Lecture S3 Muddiest Points

General Comments

Today, we derived the convolution integral (or superposition integral). This result is very important for understanding linear systems. It is used to determine the output of a system in response to an arbitrary input, when the impulse response is known.

Responses to Muddiest-Part-of-the-Lecture Cards

(28 cards)

1. ***What are examples of noncausal systems? (2 students)*** Suppose you want to do some signal processing on an audio signal. If you do the processing in real time (as the audio is occurring), the system you use must be causal — the output must depend on *past* inputs.

   Now suppose you make a recording of some data onto magnetic tape or a hard drive, and want to do signal processing on the data after the fact. The output of your signal processing system at any time can depend on the future, since the "future" is all stored on the tape or hard drive. So post-processing of data can use noncausal filters.

   Processing of digital images can be "noncausal", since the independent variable is position on the image, not time. You can use information from both the left (which is like the past) and from the right (which is like the future). I'll give an example of this in class.

2. ***Can you help us develop a physical intuition of the convolution integral? (1)*** I hope so, when we get to graphical convolution.

3. ***You said a sinusoidal function is linear. How so? (1)*** I don't think that's what I said. What I meant to say is that the oscillatory behavior that results from exciting a system with a sinusoidal response is a linear effect. That is, we can find the response of an LTI system to a sinusoid by using the convolution integral, which depends on linearity.

4. ***The math seems similar to the math in 6.041. Are the two topics related? (1)*** They are related in some ways. It is true that the probability density of the sum of two independent random variables is the convolution of their densities. So our $y(t)$, $u(t)$, and $g(t)$ are analogous to probability densities is some ways.

   There is a deep reason that both disciplines end up using the same convolution integral. Basically, it's because in both cases, we are concerned about addition on the real line. In the case of probability, it's simply because we sometimes care about the sum of two random variables with real values. In the case of signals, it's because of time-invariance (sometimes called shift-invariance), where the time that a response occurs is the time that the input occurs plus the time delay through the system.

5. ***What's a causal system? (1) Will everything we do in Unified be causal? (1)*** A system where the output of the system depends only on the past. In mathematical terms, a system is causal if $g(t) = 0$ for all $t < 0$. Most, but not all, of what we do in Unified will be causal.

6. ***Why is it referred to as the convolution integral? (1)*** "Convolution" means a form or part that is folded or coiled. (http://dictionary.reference.com/ .) As we'll see when we do graphical convolution, the integrand is performed by "folding" one signal, and then multiplying it by the other.

7. ***In the derivation, why did you divide and multiply by $\tau_{n+1} - \tau_n$ instead of $\tau_n - \tau_{n+}$? (1)*** Because $\tau_{n+1} - \tau_n$ is positive — I want $d\tau$ to be positive, not negative.

8. ***Is there a way to represent convolution graphically? (2)*** Yes. We will be doing this in lecture shortly.

9. ***If $g(t)$ is the "impulse response" and is equal to $g_s'(t)$, what does $g_s(t)$ represent? (1)*** $g_s(t)$ is the response of the system G to a unit step — it is the "step response." $g(t)$ is the response of the system to a unit impulse — it is the impulse response.

10. ***What is Duhamel's principle? (1)*** It's the result that the solution to an inhomogeneous, linear, partial differential equation can be solved by first finding the solution for a step input, and then superposing using Duhamel's integral. It's essentially what we did last time, but more specifically in a PDE framework.

11. ***Where can we find more examples? (1)*** There are examples in the notes and in the text, and we will do some in class.

12. ***Where are we going with this analysis? (1)*** We're building up a set of tools that will allow us to understand many important problems, such as modulation of radio signals, radar, control systems, etc.

13. ***If there are linear steps, why do you use the convolution instead of just adding the steps? (1)*** I'm not sure I understand. First, It's not the steps that are linear, it's the system. If the input is a sum of steps, then we can find the response as a sum of step responses, by linearity and time-invariance. By coercing $u(t)$ to look (in the limit) like a sum of steps, we find that $y(t)$ is the sum of step responses.

14. ***How many inputs are needed to solve Duhamel's integral? (1)*** Again, I'm not sure that I understand the question. Please grab me in class tomorrow.

15. ***Can your $\delta(t)$ be the same as $u(t)$? (1)*** Yes, in which case $g(t)$ is the same as $y(t)$. Recall that that $g(t)$ is the impulse response, which means that $y(t) = g(t)$ when $u(t) = \delta(t)$.

16. ***No mud. (10)*** Good. A few comments: ***I hope convolution makes more sense this term than it did in 18.03 or 6.041.*** Me too! Convolution is very important, so I try to spend enough time on it for it to make sense. ***It's too close to break to focus.*** No! This is important. You have 9 days of break — please try to focus one more day. ***More examples would be nice.*** I'll try.