# Lecture S2 Muddiest Points

## General Comments

Today we talked about Duhamel's integral, which has the nifty property of giving the response of an LTI system to an arbitrary input, $u(t)$, if we know the step response, $g_s(t)$.

I appreciate all the cards I received today. I have one request: *please* write legibly. I can't always make out what every card says.

## Responses to Muddiest-Part-of-the-Lecture Cards

(38 cards)

1. ***Who was Duhamel? (1 student)*** Jean Marie Constant Duhamel was a French mathematician who studied, among other things, heat diffusion. Heat diffusion problems are examples of LTI systems; Duhamel applied some of the ideas we talked about today to these problems.

2. ***How do you model something with steps if it's not always increasing in time? (1)*** All the math that we did works even if the slope of $u(t)$ is sometimes negative. When the slope is negative, the steps are negative (downward), which is not a problem.

3. ***The method [Duhamel's integral] seems impractical and requires a great deal of known information. Are there alternate methods? (1)*** We will be studying Laplace transform methods, which are closely related. I would disagree with you, however, that the method is impractical. It really is quite useful, and many disciplines (heat transfer, unsteady aerodynamics, etc.) use Duhamel's integral in some form.

4. ***So then the difficulty comes in modeling the step response accurately? (1)*** Yes, although this is not always that difficult. The result today says that if you can find the step response, though, you are done — you know everything there is to know about the system.

5. ***Can*** $y(t)$ ***be expressed as the following***

$$y(t) = g_s \cdot u(0)\sigma(t) + g_s \int_0^\infty \sigma(t-\tau)u'(\tau)\,d\tau$$

***(1)*** No. As you've written it, $g_s$ looks like a constant. It's not — it's a function of time. The right expression is

$$y(t) = u(0)g_s + \int_0^\infty g_s(t-\tau)u'(\tau)\,d\tau$$

6. ***It looks like Duhamel's integral has a convergence term in it. Can we use Laplace arguments to solve? (1)*** I'm not sure what a "convergence term" is, but yes, we can use Laplace methods to solve. We will do this shortly.

7. ***Could you give an example of cranking through the integral in the muddy notes? The integral looks impossible! (2)*** It's not as hard as it looks. There is a full example in the notes from the lecture today.

8. ***What is the need for the infinite upper bound on Duhamel's integral? (1)*** When we did the sum, we summed over all $\tau_n$. So the integral is, strictly speaking, over the range $\tau \in [0, \infty)$. However, examination of the integrand shows that the integrand is usually zero for $\tau > t$, since $g_s(t - \tau)$ is usually zero for $\tau > t$. (Physically, the step response is only nonzero after the step occurs.) So for most physical problems, the integral goes up to $t$. But it's not wrong to write it up to $\infty$ — it doesn't change the answer.

9. ***Suppose you were modeling a system with 55 inputs and 55 outputs, how many test cases would you have to run in the lab to determine all the step response? (1)*** There are 3025 individual step responses (one from each input to each output), but you would have to run only 55 test cases, since when you put a step input into a single input, you can measure the step response at 55 outputs simultaneously.

10. ***Is Duhamel's integral a linear transformation, kind of like Laplace's transform is a linear transform? (1)*** Yes and no. For any given step response, Duhamel's integral is a linear operator, as is Laplace's integral. However, it is an operator that takes functions of $t$ and produces functions of $t$. Laplace takes functions of $t$ (time) and produces functions of $s$ (frequency).

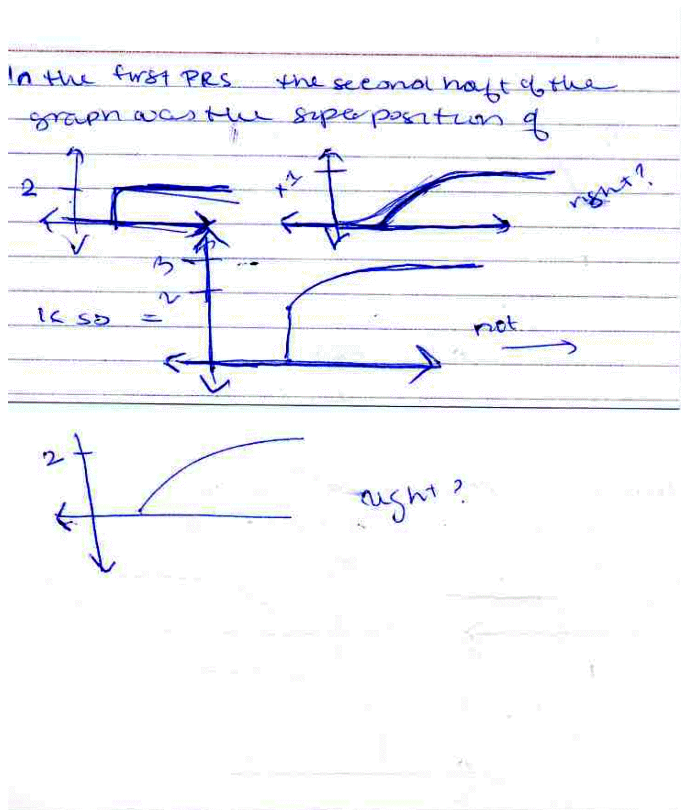
11. ***Don't understand how we went from expression of $y(t)$ using a summation series to an expression for $y(t)$ using derivatives and an integral. (1)*** In the limit as the increments in $\tau_n$ go to zero, the sum becomes an integral, just as happened in 18.01 when you approximated the area under a curve by a sum of rectangles. Also, the difference in the $u(\tau_n)$ becomes a derivative in the limit. Look this over in the notes.

12. ***If you were to find the step response experimentally, would you just have to put a value of 1 into the system and see what comes out? (1)*** That's right!

13. ***So do we know the values of $u(t)$ and $g_s(t)$ before we plug them into the formula? (1)*** Usually, yes, unless we are deriving a general result for unknown $u(t)$ and $g_s(t)$.

14. ***The most important idea today was using step functions to approximate other functions. Why did we do this? (1)*** We represent the input as a superposition of steps because we can then represent the output as a superposition of step responses. In essence, we only have to solve the differential equations of a system once (to find the step response), and we can then find any solution by an integral.

15. ***In the first PRS question, the second half of the graph was the superposition of [picture here], right. So the result is [picture], not [picture], right? (1)***



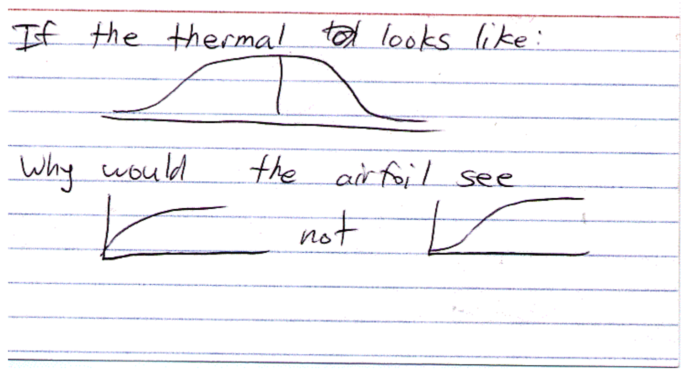

No, if I understand your question. The second part of the graph comes from the graph on the back of the card, not the front.

16. ***How do you know that the signal you are looking at is actually the step response? (1)*** Since in the lab, you have control over the input, you know whether the input is a step function or not.

17. ***In the airfoil example, the input to the system was $w(t)$ and the output was $C_L(t)$? (1)*** Yes!

18. ***Is this right? We put $u(t)$ in terms of $\sigma(t)$ and actual $u(t)$ points so that we can find $y(t)$ without having to actually find an expression for it [$y(t)$]? (1)*** Close. The point is that if we know the response to a step, then we can derive the response for any input whatsoever in terms of the step response.

19. ***If the thermal looks like [picture], why would the airfoil see [picture] not [picture]? (1)***




You're right — I didn't draw the figures carefully enough.

20. ***When you drew the response of $C_L(t)$, why does it have an inflection point? (1)*** Please look at the example in the notes. If you do the integral, the result has an inflection point.

21. ***In a real airfoil, wouldn't there be an initial value [in the lift] and then an increase in the value? (1)*** Yes. We were looking at the change in lift, which we can safely do because of linearity.

22. ***Is $u(0)g_s(t)$ a multiplication of the unit step by the unit response? (1)*** No, it is multiplication of the unit step response by the value of the input, $u(t)$, just after time zero.

23. ***No mud. (15)*** One student was unhappy about the slow pace of the lecture. There was also this question: ***what are you going to do with the index cards we turned in for the Superposition I question?*** I quickly scan through them, to get a sense of how well the class understands the concepts. Other than that, there were some nice comments: ***"Today was the day for me to fall asleep and not pay attention. But, your active learning tricks really kicked in. I especially liked having PRS [questions] that were not multiple choice. I actually felt the pressure to pay attention to everything that had happened, so that I would get the right answer. . . . You should recommend the approach to other teachers." "Whoa! Great lecture . . . you have a great way of making things make sense."*** Thanks very much!