

Lecture S11 Muddiest Points

General Comments

Today, we began looking at definitions of stability. There are many possible definitions; the definition we are using is called Bounded Input Bounded Output Stability. Basically, a system is BIBO stable if and only if nice inputs produce nice outputs. Our definition of nice is that the signals are bounded.

We got only part way through our discussion of BIBO stability. We will pick it up again in the next lecture.

Responses to Muddiest-Part-of-the-Lecture Cards

(Only 15 cards)

1. ***What is an example of a system with a bounded input and a bounded output? (1 student)*** To take a really boring example, think about one of the RC circuit examples. It's not hard to show that any of the RC circuits we looked at this year are BIBO stable. Therefore, *every* bounded input will produce a bounded output.
2. ***[Card has a drawing of a step function.] We called this bounded, but the area from zero to infinity is not finite. How is it stable? (1)*** It's not! It's true that the unit step function is bounded. However, a system which has the unit step function as its impulse response is *not* stable, because the integral (of the absolute value) is infinite. Bounded and stable are not the same thing.
3. ***What other definitions of stability are there? (2)*** Lots! There's Lyapunov stability, asymptotic stability, asymptotic stability in the large, etc.
4. ***Are these proofs four our edification, or are we going to be responsible for setting up a proof like this on the test? (1)*** From the Signals and Systems learning objectives, you should be able to "Demonstrate an understanding of the relationship between the stability and causality of systems and the region of convergence of their Laplace transforms, by correctly explaining the relationship, and using the relationship to determine the stability and causality of systems. (concept test, homework, quiz)" This is precisely the issue we are working on now.
5. ***It seems like its not really enough to assert a system is BIBO stable so long as $g(t)$ and $G(s)$ are convergent. Suppose we had a harmonic oscillator. Both g and G are convergent, but the response would blow up for a [sinusoidal] input $u(t)$. Is there something missing here? (1)*** You are right that a sinusoidal input into an undamped harmonic oscillator (with the sinusoidal frequency at the natural frequency of the oscillator) will result in an unbounded output. That fact proves that the oscillator is not BIBO stable. But that's not inconsistent with anything I said in class, because for an oscillator, the integral

$$\int_{-\infty}^{\infty} |g(t)| dt$$

is infinite. $g(t)$ is bounded, but its integral is not.

6. *The lecture was a bit confusing, maybe because you skipped some of the details like finding the limits of integration in the double integral. (1)* I'm trying to walk a fine line here, giving enough information for you to understand the concepts, without getting too lost in the mathematical details. The details are in the notes.
7. *So if we have a circuit, and we want to find $g_s(t)$, we would find $G(s)$, multiply that by $1/s$, and then take the inverse Laplace transform? (1)* Yes, exactly.
8. *Questions on [the proof done at end of lecture]. (1)* I will review next time.
9. *I'd say my understanding is unstable ... (1)* If so, we need to figure out a control law that will stabilize you. More feedback!
10. *No mud. (15)* Good.