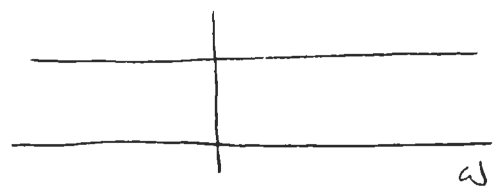
LECTURE S20

<u>Problem</u> In many types of systems (radar, navigation, etc.), we use short pulses to derive position of aircraft. Short pulses are also used to transmit digital data.

In the limit as the pulse width $\to 0$, we have an impulse. The impulse has FT (or "spectrum")

$$\mathcal{F}[\delta(t)] = 1$$



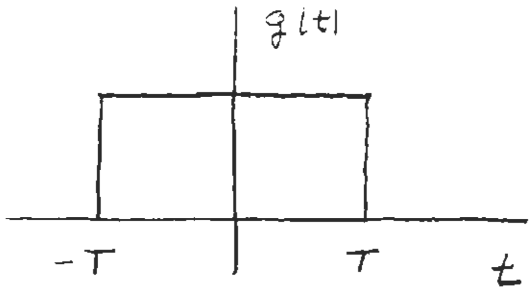
That is, a radar transmiting an impulse would broadcast at all frequencies, interfering with other channels.

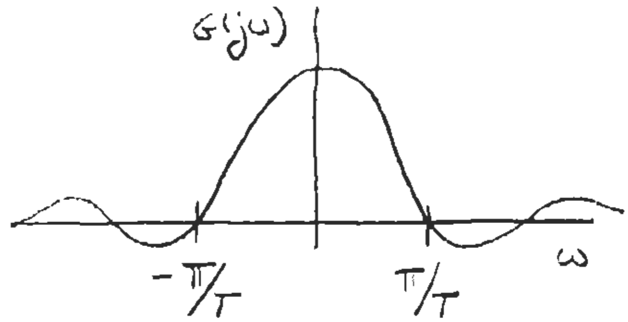
In order to reduce the <u>bandwidth</u> of the signal, we must lengthen the <u>duration</u> of the pulse. But this reduces the position resolution of the radar.

What is the relationship between duration and bandwith?

The Duration – Bandwidth Relationship

Look at a couple of transform pairs:

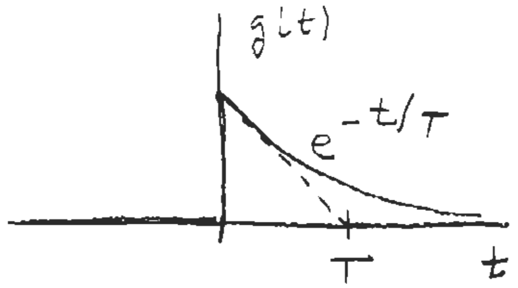


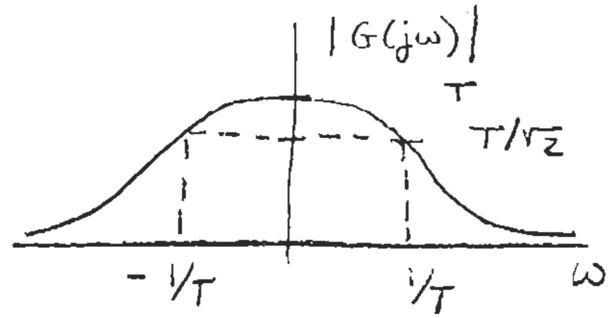


duration ~ 2T

bandwidth ~ 2π/T

duration × bandwith ~ 4π





time constant = T

half power bandwidth = 2/T

duration × bandwith ~ 2

In general,

duration × bandwidth = $(\Delta t)(\Delta \omega) \gtrsim 1$

Need definitions of duration, bandwidth that

- Conform to our rough understanding of these terms
- Can be easily computed
- Yield useful results

One useful definition is

$$\left(\frac{\Delta t}{2}\right)^2 = \frac{\int_{-\infty}^{\infty} t^2 g^2(t)\, dt}{\int_{-\infty}^{\infty} g^2(t)\, dt}$$

$g^2(t)$ ~ unnormalized probability density

$g^2(t) / \int g^2(t)dt$ ~ normalized " "

$\Delta t/2$ ~ standard deviation or radius of gyration.

Likewise,

$$\left(\frac{\Delta \omega}{2}\right)^2 = \frac{\int_{-\infty}^{\infty} \omega^2 |G(j\omega)|^2\, d\omega}{\int_{-\infty}^{\infty} |G(j\omega)|^2\, d\omega}$$

Can we relate Δt to $\Delta\omega$?

$$\left(\frac{\Delta t}{2}\right)^2 = \frac{\|t\,g(t)\|^2}{\|g(t)\|^2}$$

$$\left(\frac{\Delta\omega}{2}\right)^2 = \frac{\|j\omega\, G(j\omega)\|^2}{\|G(j\omega)\|^2}$$

But $\|G\|^2 = \|g\|^2$, by Parseval's theorem.
What is $\|j\omega G(j\omega)\|^2$?

$$j\omega\, G(j\omega) = \mathcal{F}[\dot{g}(t)]$$

$$\Rightarrow \|j\omega\, G(j\omega)\|^2 = \|\dot{g}\|^2$$

Therefore,

$$\left(\frac{\Delta t}{2}\right)\left(\frac{\Delta\omega}{2}\right) = \frac{\|t\,g(t)\| \cdot \|\dot{g}(t)\|}{\|g(t)\|^2}$$

Recall that <u>signals</u> can be viewed to be <u>vectors</u>.

$$\underline{x} \sim t\,g(t)$$
$$\underline{y} \sim \dot{g}(t)$$