Lecture S14

From last time, found that

$$\underline{x}(t) = \begin{bmatrix} v_1(t) \\ i_2(t) \end{bmatrix} = a_1 \begin{bmatrix} -\frac{1}{5} + \frac{3}{5}j \\ 1 \end{bmatrix} e^{(-1+3j)t}$$

$$+ a_2 \begin{bmatrix} -\frac{1}{5} + \frac{3}{5}j \\ 1 \end{bmatrix} e^{(-1-3j)t}$$

We want to choose a_1 and a_2 so that we match the initial conditions

$$\begin{matrix} v_1(0) = 1\,V \\ i_2(0) = 1\,A \end{matrix} \Rightarrow \underline{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\underline{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{bmatrix} -1/5 + 3/5\,j & -1/5 - 3/5\,j \\ 1 & 1 \end{bmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\Rightarrow \begin{matrix} a_1 = 1/2 - 1j \\ a_2 = 1/2 + 1j \end{matrix} \qquad \text{ugh!}$$

Therefore,

$$\underline{x}(t) = \begin{bmatrix} 0.5 + 0.5j \\ 0.5 - j \end{bmatrix} e^{(-1+3j)t}$$

$$+ \begin{bmatrix} 0.5 - 0.5j \\ 0.5 - j \end{bmatrix} e^{(-1-3j)t}$$

Use Euler's Formula to express in terms of real variables:

$$e^{\alpha + j\beta} = e^{\alpha}(\cos\beta + j\sin\beta)$$

Therefore,

$$v_1(t) = (0.5 + 0.5j)(\cos 3t + j\sin 3t)e^{-t}$$
$$+ (0.5 - 0.5j)(\cos 3t - j\sin 3t)e^{-t}$$
$$= (\cos 3t - \sin 3t)e^{-t}$$

Likewise,

$$i_2(t) = (\cos 3t + 2\sin 3t)e^{-t}$$

Note: There are "shortcuts" to reduce the math a little.

Circuits with Sources

All interesting circuits have an input source:

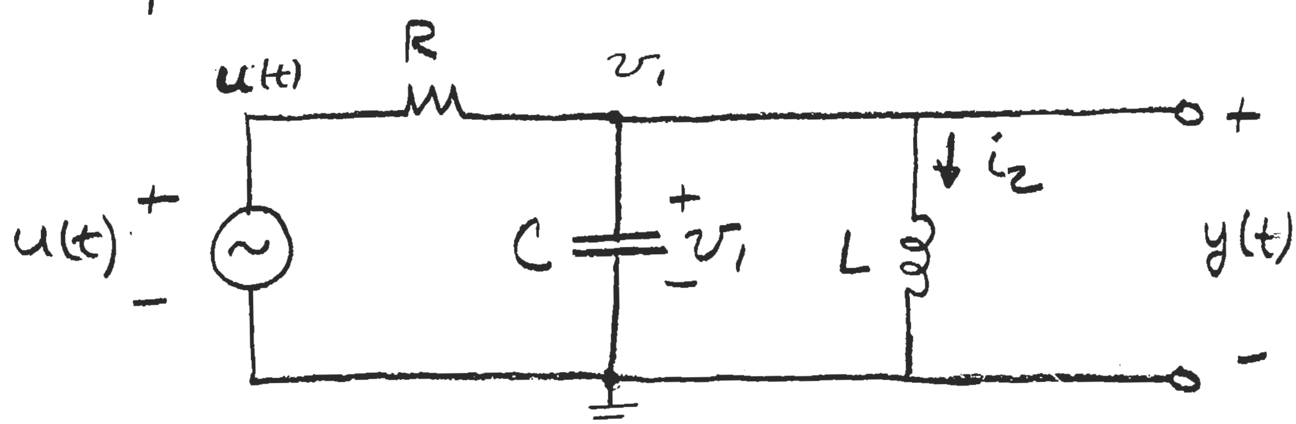
- Antenna
- Scope probe
- Tuner
- Microphone

All interesting circuits have an output:

- Loudspeaker
- Display
- Conditioned signal

We want to be able to analyze these in terms of their input/output behavior.

Example



$C = \frac{1}{2} F \qquad L = \frac{1}{5} H \qquad R = 1\Omega$

Let's find state equation:

<u>Step 1</u> States are v_1, i_2.

$$\frac{d}{dt} v_1 = \frac{1}{C} i_1$$ need to find i_1

$$\frac{d}{dt} i_2 = \frac{1}{L} v_2 = \frac{1}{L} v_C$$

To find i_1, apply KCL at v_1:

$$i_1 + i_2 + \frac{v_1 - u}{R} = 0$$

$$\Rightarrow i_1 = -\frac{v_1}{R} - i_2 + \frac{u}{R}$$

$$\Rightarrow \frac{dv_1}{dt} = -\frac{1}{RC} v_1 - \frac{1}{C} i_2 + \frac{1}{RC} u$$

So state equation, in matrix form, is

$$\frac{d}{dt}\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -1/RC & -1/C \\ +1/L & 0 \end{bmatrix}\begin{bmatrix} v_C \\ i_2 \end{bmatrix} + \begin{bmatrix} 1/RC \\ 0 \end{bmatrix} u$$

$$\dot{\underline{x}}(t) = A\,\underline{x}(t) + Bu(t)$$

The state equation in the form

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}$$

is (almost) the most general form of the dynamics equation for linear systems.

<u>Example</u> aircraft longitudinal dynamics

states: h - altitude
V - velocity
γ - flight path angle
α - angle of attack
q - pitch rate

inputs: δ_e - elevator deflection
δ_f - flap deflection
δ_T - thrust level setting

So now we have state in terms of input. What is output?

$$y = v_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = C\underline{x}$$

For some systems, can get

$$\underline{y}(t) = C\,\underline{x}(t) + D\,\underline{u}(t)$$

Again, this is the most general form.

For our example (with numbers),

$$\dot{\underline{x}} = \begin{bmatrix} -2 & -2 \\ 5 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u$$

"state eq'n."

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \end{bmatrix} u$$

"measurement eq'n"

<u>The Transfer Function</u>

Very often, we care about the response of a system to an exponential input. Why?

- Sines and cosines are exponentials, and sine waves are common signals (in testing and in practice)

$$\cos \omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$