Lecture S11

The Concept of State (continued)

Ways to think of the state:

- Each state variable is associated with "memory"

- Each state variable is associated with an initial condition.

- State variables are often associated with energy storage.

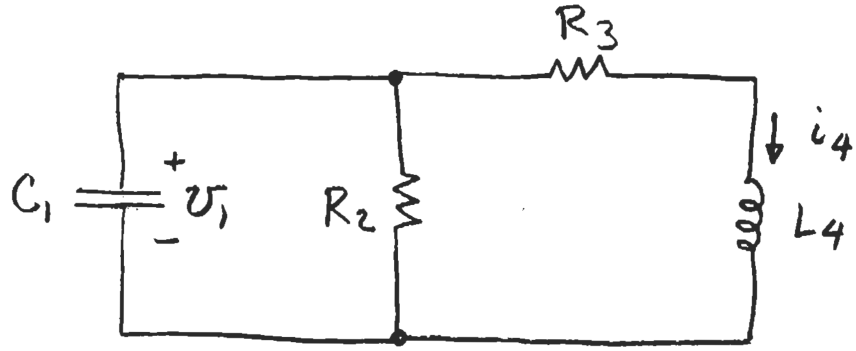
State variables are denoted by $x_1, x_2 \ldots$ and collected in the state vector $\underline{x}$:

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Goal is to find state equations of the form

$$\dot{\underline{x}}(t) = \frac{d\underline{x}(t)}{dt} = \underline{f}(\underline{x}(t))$$

Network from last time:



$C_1 = \frac{1}{2} F$

$R_2 = 4 \Omega$

$R_3 = 6 \Omega$

$L_4 = 2 H$

States of circuit are v_1, i_4.

Why?

- Energy is $\frac{1}{2} C_1 v_1^2$, $\frac{1}{2} L_4 i_4^2$
- Required initial conditions are $v_1(0)$, $i_4(0)$.
- Differential equations are

$$i_1 = C_1 \frac{dv_1}{dt} \qquad v_4 = L_4 \frac{di_4}{dt}$$

Derivative terms are indicative of states

So state vector is

$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} v_1 \\ i_4 \end{pmatrix}$$

Need to find

$$\frac{d}{dt} v_1 = f_1(v_1, i_4) \qquad \left[\dot{x}_1 = f_1(x, x_2) \right]$$

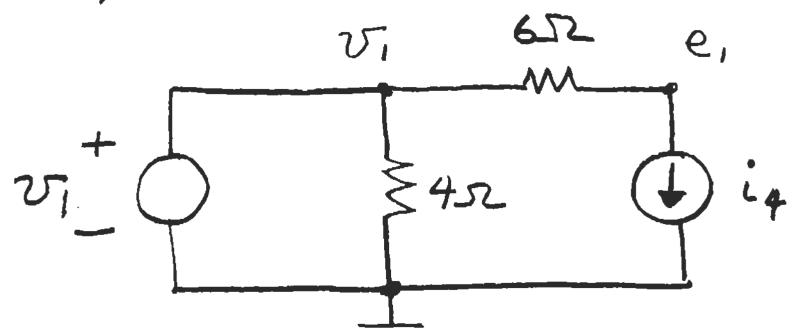
$$\frac{d}{dt} i_4 = f_2(v_1, i_4) \qquad \left[\dot{x}_2 = f_2(x_1, x_2) \right]$$

But we know that

$$\frac{d}{dt} v_1(t) = \frac{1}{C_1} i_1(t) \qquad \frac{d}{dt} i_4(t) = \frac{1}{L_4} v_4(t)$$

So need to find $i_1(t)$, $v_4(t)$ in terms of $v_1(t)$, $i_4(t)$.

Analysis circuit:



Use node method to find $e_1 = v_4$:

$$\frac{e_1}{6} - \frac{v_1}{6} + i_4 = 0$$

$$\Rightarrow \quad v_4 = e_1 = v_1 - 6\, i_4$$

$$\Rightarrow \frac{d}{dt} i_4 = \frac{1}{L_4} v_4 = \frac{1}{2} v_1 - 3 i_4$$

$$\boxed{\frac{d}{dt} i_4 = \frac{1}{2} v_1 - 3 i_4}$$

To find differential equation for v_1, need i_1. To find i_1, apply KCL at v_1 node:

$$i_1 + v_1\left(\frac{1}{4} + \frac{1}{6}\right) - \frac{1}{6} e_1 = 0$$

$$\Rightarrow i_1 = -\frac{5}{12} v_1 + \frac{1}{6}(v_1 - 6 i_4)$$

$$= -\frac{1}{4} v_1 - i_4 = C_1 \frac{dv_1}{dt} = \frac{1}{2} \frac{dv_1}{dt}$$

$$\Rightarrow \boxed{\frac{d}{dt} v_1 = -\frac{1}{2} v_1 - 2 i_4}$$

So <u>state equation</u> is

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -1/2 & -2 \\ 1/2 & -3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\underline{x}}$$

More compactly,

$$\dot{\underline{x}} = A\underline{x}$$

This form of state equation is <u>very</u> common, and <u>very</u> useful. <u>Any</u> linear, homogeneous system can be expressed in this form.

Why use this "state-space" approach?

- Very general
- There is a very rich theory describing solutions
- Approach is central to "modern" control

We care about <u>systems</u>, not just circuits. Circuits are our vehicle for understanding systems.