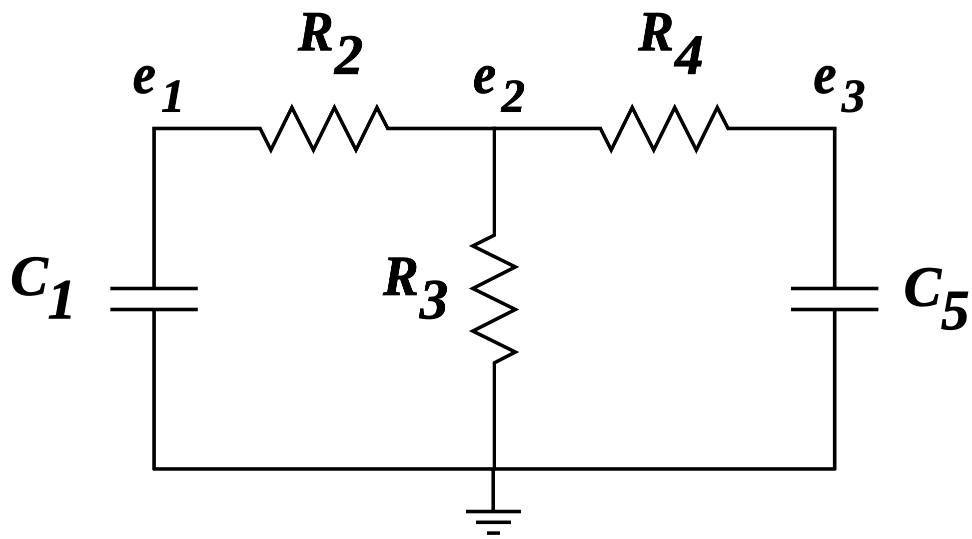
# RC Circuit Equations II




For the circuit above with $R_2 = R_3 = R_4 = 1\ \Omega$, $C_1 = C_5 = 1\ \mathrm{F}$, there is an exponential solution of the form

$$\underline{e}(t) = \underline{E}e^{st}$$

only if there is a non-trivial (i.e., nonzero) solution to the equations

$$\begin{bmatrix} s+1 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & s+1 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For what values of $s$ are there nontrivial solutions?

# RC Circuit Equations II
# Concept Test

**For what values of $s$ are there nontrivial solutions to the equations**

$$\begin{bmatrix} s+1 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & s+1 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

**My confidence in my answer is:**

1. 0%
2. 20%
3. 40%
4. 60%
5. 80%
6. 100%

# RC Circuit Equations II
# Concept Test

**There are nontrivial solutions to the equations**

$$\begin{bmatrix} s+1 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & s+1 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

**for values $s$ that satisfy the characteristic equation**

$$\phi(s) = 3s^2 + 4s + 1 = (3s+1)(s+1) = 0$$

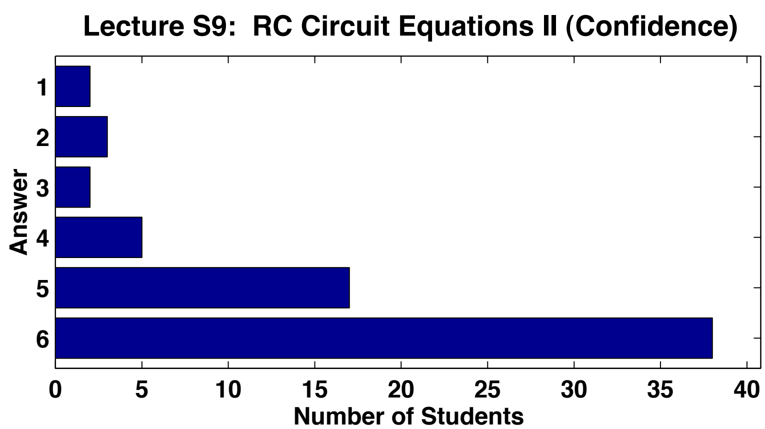
**Therefore, the characteristic values are:**

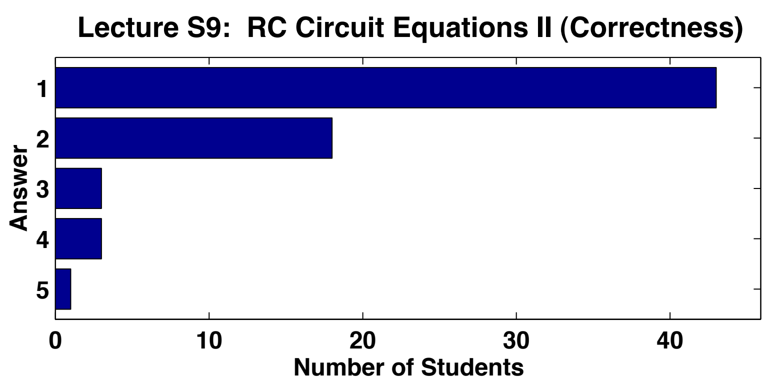
$$s_1 = -1 \text{ sec}^{-1}; \quad s_2 = -\frac{1}{3} \text{ sec}^{-1}$$

**My answer was:**

1. **Completely correct**
2. **Almost correct**
3. **Incorrect**
4. **Incomplete, but correct as far as I got**
5. **I didn’t know how to do this problem**

# RC Circuit Equations II Solution




**Most students got this one completely correct or almost correct. Good.**

# Solving for Characteristic Vectors Concept Test

**Find a solution to the equation**

$$\begin{bmatrix} 2/3 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2/3 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

**My confidence in my answer is:**

1. 0%
2. 20%
3. 40%
4. 60%
5. 80%
6. 100%

# Solving for Characteristic Vectors Concept Test

**A solution to the equation**

$$\begin{bmatrix} 2/3 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2/3 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

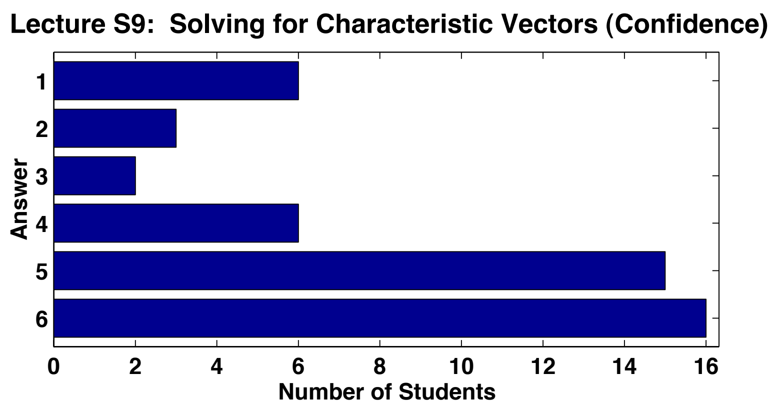
**is**

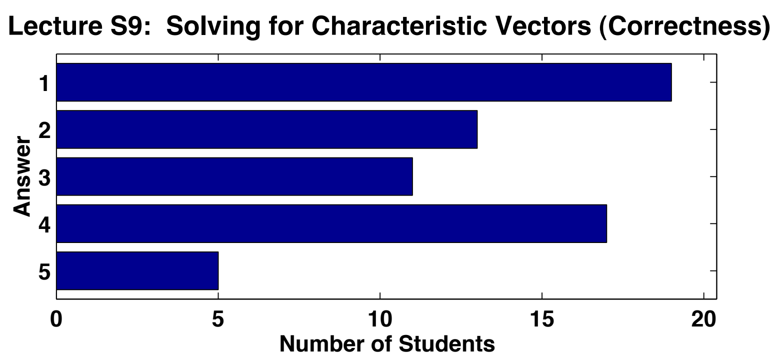
$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2/3 \\ 1 \end{bmatrix}$$

**My answer was:**

1. **Completely correct**
2. **Almost correct**
3. **Incorrect**
4. **Incomplete, but correct as far as I got**
5. **I didn't know how to do this problem**

# Solving for Characteristic Vectors Solution




**Although many students were able to solve by inspection, the more reliable approach is to do row reduction. The original matrix is:**

$$\begin{bmatrix} 2/3 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2/3 \end{bmatrix}$$

**Normalize first row:**

$$\begin{bmatrix} 1 & -3/2 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2/3 \end{bmatrix}$$

**Eliminate -1 from first column in second row by subtracting -1 times the first row:**

$$\begin{bmatrix} 1 & -3/2 & 0 \\ 0 & 3/2 & -1 \\ 0 & -1 & 2/3 \end{bmatrix}$$

**Normalize the second row:**

$$\begin{bmatrix} 1 & -3/2 & 0 \\ 0 & 1 & -2/3 \\ 0 & -1 & 2/3 \end{bmatrix}$$

**Eliminate -1 from second column in third row by subtracting -1 times the second row:**

$$\begin{bmatrix} 1 & -3/2 & 0 \\ 0 & 1 & -2/3 \\ 0 & 0 & 0 \end{bmatrix}$$

**Can then arbitrarily choose $E_3 = 1$. Then the second row means**

$$E_2 - (2/3)E_3 = 0$$

**So $E_2 = 2/3$. Continuing the back substitution, $E_1 = 1$.**