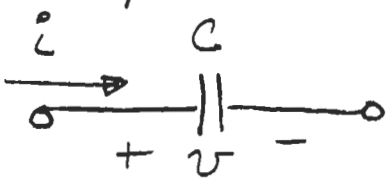
Lecture 57

Energy Storage Elements

- Resistive networks are <u>static</u>.
  Voltages and currents don't change with time, if sources are constant.
- Systems with energy storage elements are <u>dynamic</u>
  Voltages and currents depend on time history of source strengths

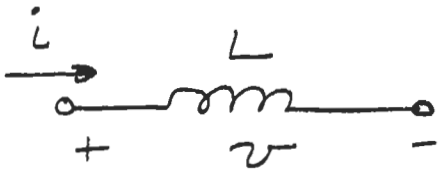
Energy storage elements:

Capacitor:



$$i = C\frac{dv}{dt}$$

Inductor:



$$v = L\frac{di}{dt}$$

Capacitors resist voltage change — requires current flow to change voltage

Inductors resist current change — requires voltage to change voltage

How much energy is stored?

$$\text{Energy} = \int_{-\infty}^{t} \text{Power}\, d\tau$$

$$= \int_{-\infty}^{t} i(\tau)\, v(\tau)\, d\tau$$

$$= \int_{-\infty}^{t} C \frac{dv(\tau)}{d\tau} v(\tau)\, d\tau$$

$$= \frac{1}{2} C v^2(t)$$

$$E = \frac{1}{2} C v^2(t) \quad \text{for capacitor}$$

Similarly,

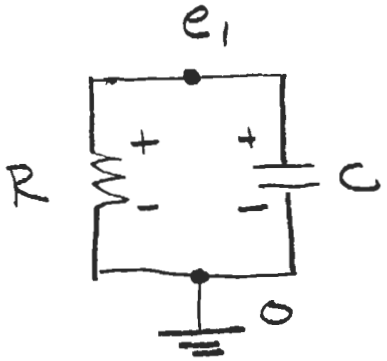
$$E = \frac{1}{2} L i^2(t) \quad \text{for inductor}$$

Our goal is to find time response of circuits with inductors and capacitors.

Roadmap:

- ☐ RC networks
  - easily solved using node method
- ☐ RL networks
  - easily solve using loop method
- ☐ RLC networks
  - Hard to solve using either method
  - Solve by <u>impedance</u> or <u>state</u> methods

Simple RC network:



Solve by node method

KCL at node e_1:

$$0 = i_R + i_C$$

$$= \frac{e_1 - 0}{R} + C\frac{d}{dt}(e_1 - 0)$$

$$= C\frac{d}{dt}e_1 + \frac{1}{R}e_1 = 0$$

$$\Rightarrow \boxed{\dot{e}_1(t) + \frac{1}{RC}e_1(t) = 0}$$

Solution is an <u>exponential</u>:

$$e_1(t) = e_1(0)\, e^{-t/RC}$$

(check by plugging in). Will talk more about how to solve later.