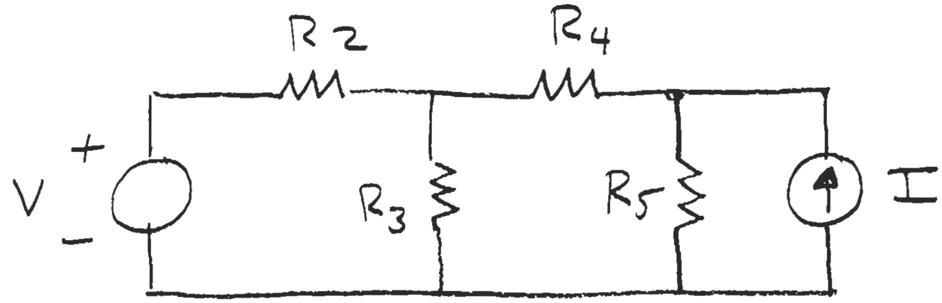
Lecture S4

The Node Method

Consider the network:

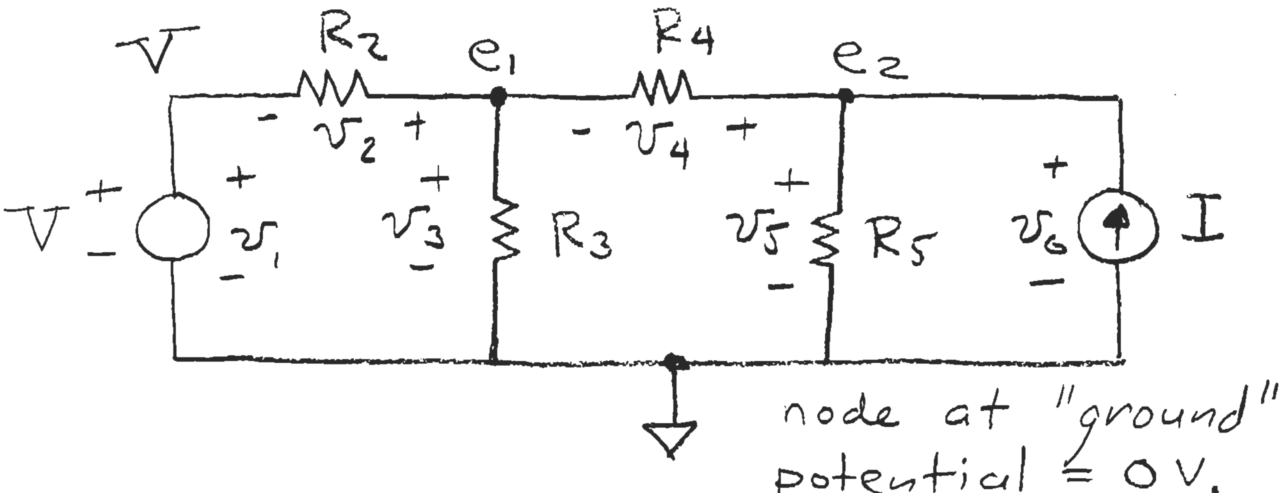


This simple network has

6 branch currents	3 (independent) loops
6 branch voltages	3 (independent) nodes
	6 constitutive laws
12 unknowns	12 equations

So can solve, but what a mess!
Need an organized way to solve —
the node method!

Key observation —
KVL holds for networks because we can assign a potential to each node, and branch voltages are just differences in potentials



KVL in center loop:

$$-v_3 - v_4 + v_5$$

$$= -(e_1 - 0) - (e_2 - e_1) + (e_2 - 0)$$

$$= 0$$

So don't have to worry about KVL!

Now apply KCL at nodes.

e_1: $\quad i_2 + i_3 - i_4 = 0$

$$\Rightarrow \frac{1}{R_2}(e_1 - V) + \frac{1}{R_3}(e_1 - 0) - \frac{1}{R_4}(e_2 - e_1) = 0$$

Collecting terms:

$$\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)e_1-\frac{1}{R_4}e_2-\frac{1}{R_2}V=0$$

Let $G=\frac{1}{R}$. ($G =$ "conductance") Then

$$(G_2+G_3+G_4)e_1-G_4e_2=G_2V$$

[do node method concept test]

At node e_2, current out of node is

$$\frac{e_2-e_1}{R_4}+\frac{e_2-0}{R_5}-I=0$$

$$\Rightarrow \left(\frac{1}{R_4}+\frac{1}{R_5}\right)e_2-\frac{1}{R_4}e_1=I$$

$$\Rightarrow (G_4+G_5)e_2-G_4e_1=I$$

So the two equations together are:

$$\begin{cases} (G_2 + G_3 + G_4)\, e_1 - G_4 e_2 = G_2 V \\ -G_4 e_1 + (G_4 + G_5)\, e_2 = I \end{cases}$$

In matrix form, the equations are:

sum of conductances connected to e_1 — conductance connecting e_1 to e_2 — conductance × Voltage of Known source connected to e_1

$$\begin{bmatrix} G_2 + G_3 + G_4 & -G_4 \\ -G_4 & G_4 + G_5 \end{bmatrix} \begin{Bmatrix} e_1 \\ e_2 \end{Bmatrix} = \begin{Bmatrix} G_2 V \\ I \end{Bmatrix}$$

conductance connecting e_2 to e_1 — sum of conductances connected to e_2 — Known sour.. into e_2

Can solve using Cramer's rule or Gaussian elimination:

$$e_1 = \frac{\begin{vmatrix} G_2 V & -G_4 \\ I & G_4 + G_5 \end{vmatrix}}{\begin{vmatrix} G_2 + G_3 + G_4 & -G_4 \\ -G_4 & G_4 + G_5 \end{vmatrix}}$$

$$= \frac{(G_4 + G_5) G_2 V + G_4 I}{(G_4 + G_5)(G_2 + G_3 + G_4) - G_4^2}$$

$$e_2 = \frac{(G_2 + G_3 + G_4) I + G_2 G_4 V}{(G_4 + G_5)(G_2 + G_3 + G_4) - G_4^2}$$

(In a real problem, would be much easier to do numerically than symbolically.)

[second concept test here]