Lecture S3

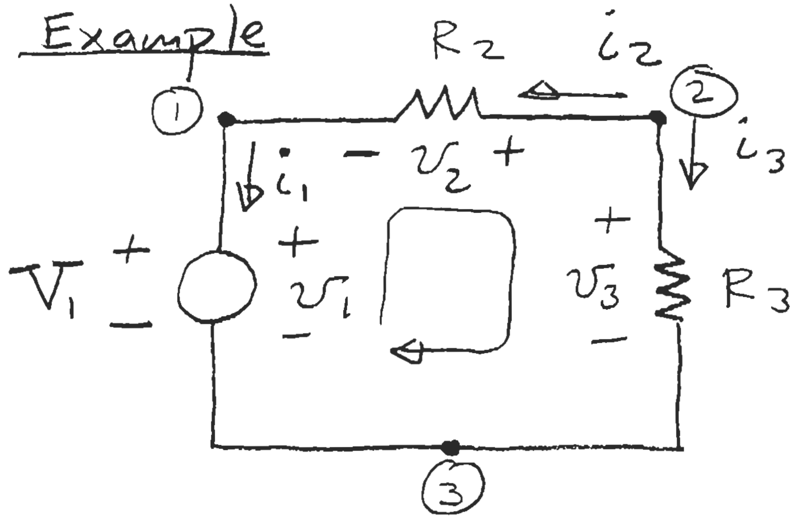
Kirchhoff's Voltage Law

In any circuit, the algebraic sum of voltages around any loop is zero.

This is a result of conservation of energy. If I take a test charge around the loop, it should have no net change in energy.

Said another way, we know there is a unique electric potential at each node. The branch voltage of each element is the difference in potential across the element. Summing the differences around the loop must yield zero (see node method).

Example



Apply KVL to (the only) loop:

$$-v_1 - v_2 + v_3 = 0 \qquad (1)$$

This is the sum of voltage <u>drops</u>. Just take first sign you get to.

<u>Kirchhoff's Current Law</u>

The sum of currents flowing <u>out of</u> each node is zero.

This follows from conservation of charge, and the fact that charge can't accumulate at a node.

Apply to example:

@ node ①: $i_1 - i_2 = 0 \qquad (2)$

@ node ②: $i_2 + i_3 = 0 \qquad (3)$

@ node ③: $-i_1 - i_3 = 0 \qquad (4)$

We have 3 homogeneous linear equations in 3 unknowns, so we should be able to solve. The result is

$i_1 = i_2 = i_3 = 0$ (WRONG!)

What's wrong with this conclusion?

Note that equation (4) is redundant:

$$(4) = -(2) - (3)$$

So (4) has no additional information. So we really have <u>2</u> equations in 3 unknowns.

<u>Constitutive Laws</u>

Finally, we apply the constitutive laws:

$$v_1 = V_1 \qquad (5)$$
$$v_2 = i_2 R_2 \qquad (6)$$
$$v_3 = i_3 R_3 \qquad (7)$$

So we have 6 independent equations [(1), (2), (3), (5), (6), (7)] in 6 unknowns, and we can solve.

Work on currents first:

$$(2) \Rightarrow i_1 = i_2$$

$(3) \Rightarrow \quad i_2 = -i_3$

So set

$$i_1 = -i$$
$$i_2 = -i$$
$$i_3 = i$$

Now apply constitutive laws to (1):

$$0 = -v_1 - v_2 + v_3$$

$$= -V_1 - i_2 R_2 + i_3 R_3$$

$$= -V_1 + i R_2 + i R_3$$

$$\Rightarrow \quad i = \frac{V_1}{R_2 + R_3} = i_3 = -i_2 = -i_1$$

$$v_1 = V$$

$$v_2 = i_2 R_2 = -\frac{R_2}{R_2 + R_3} V$$

$$v_3 = i_3 R_3 = \frac{R_3}{R_2 + R_3} V$$

whew!!

This is a "voltage divider." It divides the voltage between R_2 and R_3, in proportion to their resistances.

The approach works, but

- Solution approach was not very organized
- There were redundant equations
- It would be very difficult for a larger circuit.