Notes on the Node Method and the Loop Method

The steps in the node method are:

1. Identify one node as the ground node, so that by definition, the potential at the node is 0 V. You may choose any node as ground, but often a judicious choice will simplify things later on. If there is only one source, make the negative terminal of the source ground. If there are several sources, all with a common node, make that node ground. Otherwise, choose one source, and make one of its terminals (usually the negative terminal) ground.

2. Label the potential of each node. Of course the ground node is at 0 V. Most of the other nodes will be unknown. Label them as e_1, e_2, etc. For voltage sources, use the constitutive relation to label one of the nodes. For example, if the negative terminal is at ground, and the source has strength V_1, then the positive terminal is at V_1. If the negative terminal is at, say, e_2, then the positive terminal is at $V_1 + e_2$.

   Generally, this process will lead to a unique (but perhaps unknown) voltage at each node. Furthermore, Kirchhoff's voltage law will be satisfied automatically for each loop.

   This process can fail in one situation: If any loop in the network consists of only voltage sources, then that loop will not satisfy KVL (unless the source strengths happen to sum to zero around the loop). Physically, such a situation would lead to infinite current flow, and so should be avoided!

3. For each node with unknown potential, apply Kirchhoff's Current Law. This will lead to an equation in the unknown node voltage. (The equation will also involve other nodes that are connected to the node of interest by other elements.)

   There is no need to apply KCL at nodes with known voltage. Indeed, such nodes are connected to voltage sources, and the constitutive relation of voltage sources gives no information about the current flow through the source; hence, it adds no new information that would allow one to find the unknown node voltages.

   There is one complicated situation that bears discussion. In cases where both nodes connected to a voltage source is unknown, because neither terminal is at ground, there will be one unknown associated with the two nodes. For example, the negative terminal of the source may be at e_2, and the positive terminal at $V_1 + e_2$. In that case, KCL is applied to the two nodes together. That is, the two nodes are treated as a *supernode*, and all the current flowing out of *both* nodes is summed. This will yield one equation for the one unknown.

4. The resulting linear equations are solved for the unknown node voltages.

5. If the currents through the various network elements are desired, the constitutive relations are used to derive them. In the case of voltage sources, KCL must be applied at one node of the source to find the source current.

The steps in the loop method are:

1. Identify the maximum number of independent loops in the network. If the loop is *planar*, i.e, has no crossovers of elements or conductors, then the number of independent loops is obvious — it's the number of "white spaces" the circuit encloses.

   If the circuit is not planar, it's a little more tricky. The number of independent loops is $b - n + 1$, where b is the number of branches or elements in the circuit, and n is the number of nodes. You must pick loops that are really independent. That is, you must be able to build up any set of branch currents that is consistent with KCL from the loop currents. See Appendix A for more details.

2. Label the current around each loop, with known currents where possible (due to current sources), and otherwise with unknown variables.

   It is important to understand what is meant by "loop current." The current through any element is given by the algebraic sum of loop currents of the loops that that element is part of. As such, the loop current is simply a bookkeeping convenience that will be useful for the method.

   Generally, this process will lead to a unique (but perhaps unknown) current associated with each loop. Furthermore, Kirchhoff's current law will be satisfied automatically at each node.

   This process can fail in one situation: If any node in the network is connected only to current sources, then that node will not satisfy KCL, unless the source strengths happen to sum to zero. Physically, such a situation would lead to infinite voltage at that node, and so should be avoided!

3. For each loop with unknown current, apply Kirchhoff's Voltage Law. This will lead to an equation in the unknown loop current. (The equation will also involve other loop currents, namely, for other loops which share a common element.)

   There is no need to apply KVL for loops with known current. Indeed, such loops include current sources, and the constitutive relation of current sources gives no information about the voltage across the source; hence, it adds no new information that would allow one to find the unknown loop currents.

4. The resulting linear equations are solved for the unknown loop currents.

5. If the voltage across the various network elements are desired, the constitutive relations are used to derive them. In the case of current sources, KVL must be applied to the loop that includes the source to find the source voltage.

Appendix A. Number of Independent Loops

This brief set of notes clarifies an issue that arose in class, namely, When using the loop method, how many independent loops can be identified? And, how do we know that a set of loops is independent?

Consider the circuit shown in Figure 1. The circuit has $n = 5$ nodes, labeled e_1,

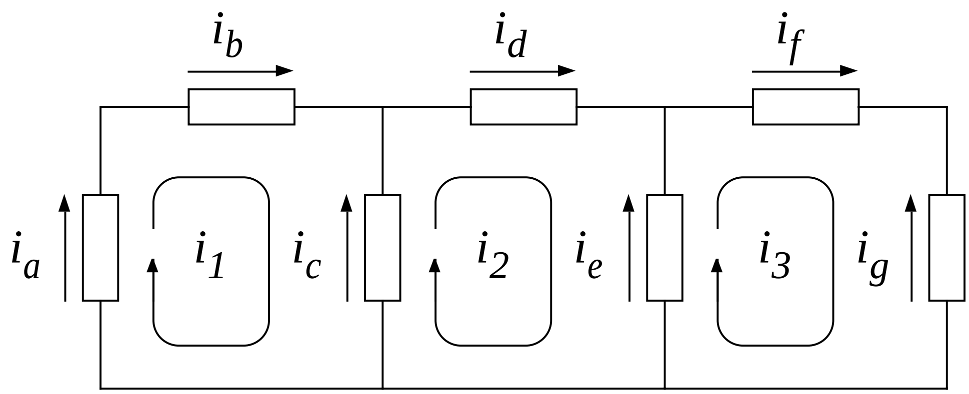

Figure 1: Example circuit

$e_2, \ldots e_5$, and $b = 7$ branch currents, corresponding to the 7 elements in the circuit. Also shown in the figure are three loop currents, i_1, i_2, and i_3. It's important to understand that the loop currents are used to describe the branch currents in the circuit. For example, the current i_c is given in terms of loop currents by

$$i_c = -i_1 + i_2 \tag{1}$$

Kirchhoff's current law applies at each node. KCL for this network can be written in vector form as

$$\begin{aligned} \underline{0} &= \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_d \\ i_e \\ i_f \\ i_g \end{bmatrix} \\ &= A\underline{i} \end{aligned} \tag{2}$$

Note that the matrix A not only describes KCL at the five nodes, it also encodes the topology of the network. For example, the first column of A shows that the circuit element a is connected between node e_5 and node e_1.

Now, each column of A is composed of all zeros, except for one 1 and one -1, meaning that each element is connected to two nodes of the network. Thus, each column of A sums to zero, so that even though A is a 5×7 matrix, one row is redundant, so that there are only four independent rows. Since there are seven unknown currents, the equation above is under determined, and there are many solutions to the equations. In fact, the space of solutions is a 3-dimensional space. The general solution to Equation (2) is given by

$$\underline{i} = B \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} \tag{3}$$

where B is a 7×3 matrix, the three columns of B are linearly independent, and

$$AB = 0 \tag{4}$$

By choosing the currents in this way, we automatically guarantee that KCL is satisfied at each node, since each loop current (*i.e.*, column of B) satisfies KCL.

One choice of B is

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \tag{5}$$

This choice corresponds to the three loop currents shown in Figure 1. For example, the first column of B corresponds to the loop current i_1.

So how many independent loops does a circuit have? For a circuit with n nodes, there are $n-1$ independent node equations that constrain the currents. If the circuit has b branches, that implies that there are exactly $b-(n-1) = b-n+1$ independent loops currents. For planar circuits (circuits that can be drawn on a plane without any elements crossing one another), there is an obvious choice for the loops; namely, the $b-n+1$ small loops that enclose no circuit elements. For nonplanar loops, the choice is not so obvious, but the number of independent loops will be $b-n+1$.