

## Lecture S14 Muddiest Points

### General Comments

We wrapped up the last Signals and Systems lectures by talking about transfer functions, which describe the ratio of the output amplitude to the input amplitude, for sinusoidal inputs. We saw that we can compute the transfer function from the state-space description, or by using impedance methods. Of course, the two approaches give the same results.

We applied these results to a bandpass filter, and showed how a bandpass filter can selectively filter out all frequencies except for those in a narrow band of frequencies.

And, of course, we had doughnuts.

### Responses to Muddiest-Part-of-the-Lecture Cards

(41 cards)

1. *If the transfer function is*

$$G(s) = C(sI - A)^{-1}B + D \quad (38)$$

*and  $(sI - A)^{-1}$  is a matrix, how is it that  $G(s)$  is just a scalar function? (2 students)* If  $A$  is  $n \times n$ ,  $B$  is  $n \times 1$ ,  $C$  is  $1 \times n$ , then the product  $C(sI - A)^{-1}B$  is a scalar.

2. *How did you find*

$$y(t) = [1 \quad 0] \begin{bmatrix} v_1 \\ i_2 \end{bmatrix} + [0] u(t) \quad ? \quad (39)$$

(3) Because the output terminals connect to the same nodes as the capacitor,

$$y(t) = v_1 \quad (40)$$

This is the same as

$$y(t) = [1 \quad 0] \begin{bmatrix} v_1 \\ i_2 \end{bmatrix} + [0] u(t) \quad ? \quad (41)$$

Although this seems to add complication, the most general form really is

$$y = C\underline{x} + Du \quad (42)$$

since the voltage across any two nodes is a linear combination of the source strengths.

3. *How did you find the transfer function from the impedances? (2)* I treated the impedances as resistors, and solved for  $Y$  in terms of  $U$  and the impedances. The result is of the form  $Y = G(s)U$ .
4. *Significance of impedance? (1)* See above. Capacitors and inductors act a little like resistors for sinusoidal signals. By using impedances, we can use the algebra we used to solve resistive networks to solve differential equations.

5. **Will we be given  $G(s)$  for a system, or will we have to derive it? (1)** From the learning objectives, “Students graduating from 16.010/020 will be able to: Derive the transfer function for RLC networks and other dynamic systems, using state methods.” You might also be able to derive the transfer function using impedance methods.
6. **Will we be doing state functions symbolically on the quiz? (1)** Perhaps. See above.
7. **Second half of the lecture was muddy. What is it that we were doing exactly? (1)** We were trying to understand how systems (circuits in this case) respond to sinusoidal inputs. We found that the output is sinusoidal, with the ratio of output to input amplitude described by the transfer function.
8. **For the bandpass filter, how does  $R$  affect the width of the graph? (1)** The larger  $R$  is, the more narrow the graph.
9. **Unclear about the concept of superposition. (1)** For a linear circuit, the response of the circuit to a number of sources is the sum (superposition) of the response of the circuit to each of the sources alone. For a linear differential equation, if the input is  $u(t) = \sum u_i(t)$ , and the response to each  $u_i(t)$  is  $y_i(t)$ , the response to  $u(t)$  is  $y(t) = \sum y_i(t)$ .
10. **Can you give an example of how to invert a  $2 \times 2$  matrix? (1)** Sure. The inverse of a general  $2 \times 2$  matrix is

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad (43)$$

For a larger matrix, the formula is more complex.

11. **So the  $s$  in the  $G(s)$  is the  $j\omega$  in the input? (1)** Yes. If the input is  $e^{j\omega t}$ , then  $s = j\omega$ .
12. **Can we do a whole example on finding the transfer function with values? (1)** There is an example in the notes, and you have a problem on the problem set.
13. **I don't understand  $|G(j\omega)|$  at all. (1)**  $G(j\omega)$  is, in general, a complex function of  $\omega$ . The magnitude of  $G(j\omega)$  is the ratio of the amplitude of the output sinusoid to the amplitude of the input sinusoid. The phase of  $G(j\omega)$  gives the phase difference between the input and output sinusoids.
14. **Will we only be calculating  $G(s)$  for a bandpass filter? (1)** No. On the problem set, you will calculate  $G(s)$  for a notch filter.
15. **Muddy: Derivation of  $G(s)$ . (1)** Understood. Please see the notes, or see me at office hours.
16. **Do we need to conceptually understand bandpass filters for the quiz. (1)** Sure, to the level we discussed in class today.
17. **No mud. (21)** Lots of very nice comments: “That transfer function derivation was fascinating!” “Great lectures! I really enjoyed your class!” “I have really enjoyed Signals and Systems! Thank you for lecturing us Prof. Hall!” Thanks! It’s been a good term — I’m looking forward to continuing this next term.