Lecture S12 Muddiest Points

General Comments

In this lecture, we continued our discussion of states. We started with a concept tests on the number of states required in modelling a roller coaster, which caused some confusion. We continued with a description of a circuit in state-space form, and found the eigenvalues of the system, which are the same as the characteristic values.

Responses to Muddiest-Part-of-the-Lecture Cards

(44 cards)

1. ***The first PRS question was worded unclearly. (1)*** Sorry if you found it to be unclear. Was the wording unclear, or was it difficult to figure out what the states are?
2. ***In the PRS question, no one knew you could assume you could assume you know the velocity distribution along the track. (1)*** I don't think I said that. I said that you know that the velocity vector is along the track. Is that what you mean? If so, that was part of the point of the problem — to determine what the minimum number of initial conditions is.
3. ***identifying states — not sure how to do it yet. (1)*** For circuits, it's easy — it's all the capacitor voltages and all the inductor currents. For other systems, it takes a little more work. The easiest way to identify them is to figure out what initial conditions are required. These correspond to states.
4. ***The RLC circuit is pretty much analogous to the roller coaster, right? (1)*** Exactly! In both cases, two states are needed to describe the system.
5. ***For the concept question, the only states are d and v, because both can be determined from known parameters of s? (1)*** d and v are states, because they are the only initial conditions you need to determine the future behavior of the coaster.
6. ***For the concept question, will the same states apply to an inverted coaster? (1)*** Yes, so long as the coaster is constrained not to leave the rails.
7. ***What are the differences between a state and a parameter? It seems that some of the parameters you stated are time-varying. (1)*** In the roller coaster question, the parameters are functions of the state, not time. So you can always determine the time derivatives of the state by knowing the state, and the geometry of the coaster. Parameters can vary with time (implicitly, through their dependence on the states, or explicitly). The point is that the geometry of the coaster is known *a priori*, and hence the geometry is a parameter.
8. ***Why does this (state) method for solving dynamic circuits seem different from what we've done before? Is it different? (1)*** In one sense, it's not different. We're applying KVL, KCL, and the constitutive laws to find the dynamic equations. In another,

we are requiring that we find the equations of motions in a special form. So what we're doing is kind of a special case.

9. ***Could you explain solving the equation with eigenvalues? (1)*** The steps are: (1) Find the determinant of $sI - A$. (2) Solve for the values of s so that $\det(sI - A)$. The PRS equation is worked out in the notes.

10. ***Can we go over the signs on your eigenvectors on the last PRS question?*** I'm not sure what you mean. Please ask at office hours or in recitation.

11. ***Will we ever end up using 4×4 matrix? (1)*** If you ever have a system with 4 states, you will need a 4×4 matrix for the state matrix. We probably won't need a matrix this large in class, but you certainly would in real life.

12. ***Can you give some examples of $\underline{x}$ and $\dot{\underline{x}}$ in terms of currents and voltages? I'm having trouble following how the equations for i and v of the form $\dot{\underline{x}} = A\underline{x}$. (1)*** In class, we found that

$$\frac{dv}{dt} = a_{11}v + a_{12}i \tag{22}$$

$$\frac{di}{dt} = a_{21}v + a_{22}i \tag{23}$$

(The a's are constants that depend on the circuit diagram.) We identify the state vector as

$$\underline{x} = \begin{bmatrix} v \\ i \end{bmatrix} \tag{24}$$

and the matrix A as

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \tag{25}$$

13. ***Why do we need to take eigenvalues and eigenvectors? What does this tell us about the circuit? (1)*** Eigenvalues and eigenvectors tell us how a circuit (or any linear, homogeneous, time-invariant system) evolve over time.

14. ***How would you define "eigenvalue" and "eigenvector"? (1) How do you get the eigenvectors after you get the eigenvectors? (1)*** Eigenvalues are the roots of $\det(sI - A) = 0$. An eigenvector is the solution to the equation $(s_1 I - A)\underline{X} = \underline{0}$, where s_1 is an eigenvalue.

15. ***Class is too early. Also, we haven't even mentioned eigenvalues in 18.03. (1)*** (a) 10 a.m. is too early? Most people start work at 8 a.m. (b) I know you haven't seen eigenvalues yet in 18.03. That's why I'm showing you all you need to know. In 18.03, they will also tell you that the eigenvalues are the roots of $\det(sI - A) = 0$. Also, you've been doing eigenvalues, effectively, when we found characteristic values and vectors.

16. ***How do states apply to circuits? (1)*** Like most dynamic systems, circuits can be described using state equations. Circuits have states, airplanes have states, satellites have states. We're solving circuits, but we're also learning how to deal with general systems.

17. ***Can you explain how***

$$sI - A = \begin{bmatrix} s+1/2 & 2 \\ -1/2 & s+3 \end{bmatrix}$$

(1) We found that

$$\frac{d\underline{x}}{dt} = A\underline{x} \tag{26}$$

We then substituted the trial solution

$$\underline{x}(t) = \underline{X}e^{st} \tag{27}$$

to obtain

$$s\underline{X} = A\underline{X} \tag{28}$$

The left hand side is a scalar times a vector. The right hand side is a matrix times a vector. To make the left side look more like a right, we write $s\underline{X} = sI\underline{X}$, where I is the identity matrix

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{29}$$

Moving the right hand side to the left,

$$sI - A = 0 = \begin{bmatrix} s+1/2 & 2 \\ -1/2 & s+3 \end{bmatrix}$$

18. ***Are there notes on how to find eigenvalues online? (1)*** Not by me. The 18.03 text is a good place to start. See the Signals and Systems main page for the reading.

19. ***Muddy: How to find eigenvalues. (1) More eigen examples! (with smiley) (1)*** Their will be one in the homework, and we will do some in recitation.

20. ***What are the values for the A matrix in the $(sI - A) = 0$ equation? (1)*** In a system with, say, two states, x_1 and x_2, you first find the differential equations of x_1 and x_2. If the system is linear and homogeneous, the equations will have the form

$$\begin{aligned} \frac{dx_1}{dt} &= a_{11}x_1 + a_{12}x_2 \\ \frac{dx_2}{dt} &= a_{21}x_1 + a_{22}x_2 \end{aligned} \tag{30}$$

Then the matrix A is

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \tag{31}$$

21. ***Too much 18.03 (1)*** How so?

22. ***No mud. (21)*** Good! A few comments. ***I am sleepy.*** I'm not sure how I can help you with that problem. It is true that you will learn better if you aren't too sleepy, though. ***As always, very clear lecture.*** Thanks very much! ***Don't oversimplify so much and talk down to us with concept questions directions.*** I'm sorry if I offended you. I was trying to get folks to defend their solutions to one another, because I knew (from prior years) that this is a tricky one for students.