

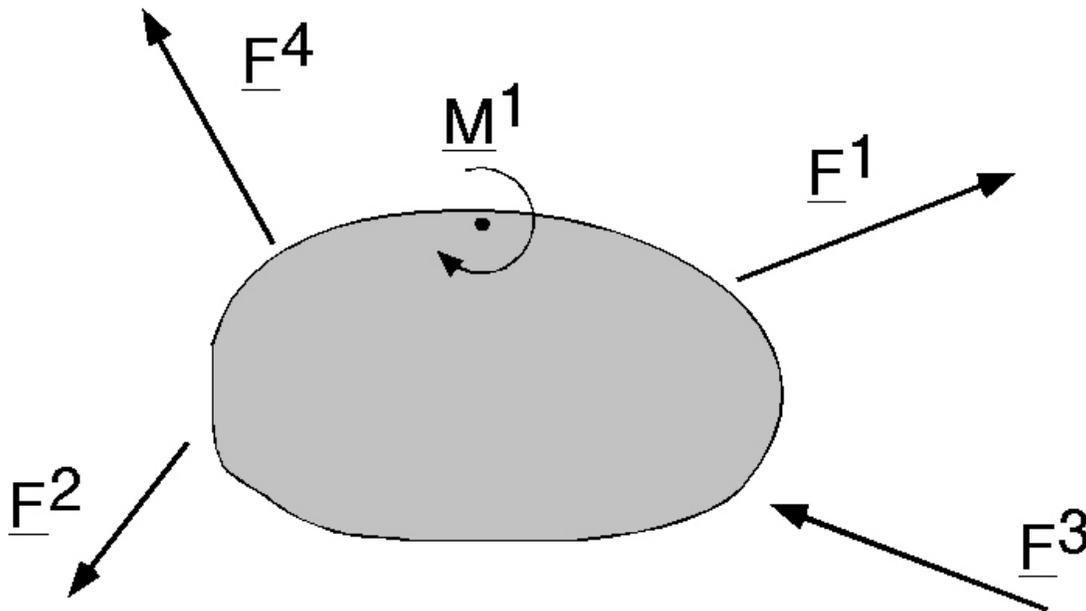
Lecture M3 More about EquilibriumEquilibrium of a Body

A body can be considered to be a system of particles. In the limit, a body is an infinite set of infinitesimal masses (particles).

So we can generalize the equations of equilibrium into two equations:

$$\sum \underline{F}^{(n)} = 0 \quad \text{Equilibrium of forces}$$

$$\sum \underline{M} + \sum \underline{r}^{(n)} \times \underline{F}^{(n)} = 0 \quad \text{Equilibrium of moments}$$



- Only applies for static equilibrium (otherwise $\sum \underline{F} = m\underline{a}$, $\sum \underline{M} + \sum \underline{r}^{(n)} \times \underline{F}^{(n)} = I\dot{\omega}$)
- Deals with external forces only
- Gives two necessary and sufficient conditions for equilibrium (6 scalar equations total)

Now consider concepts in two dimensions (2-D) so that we can more easily represent comprehend them, but do not lose track of the 3-D generality of these ideas:

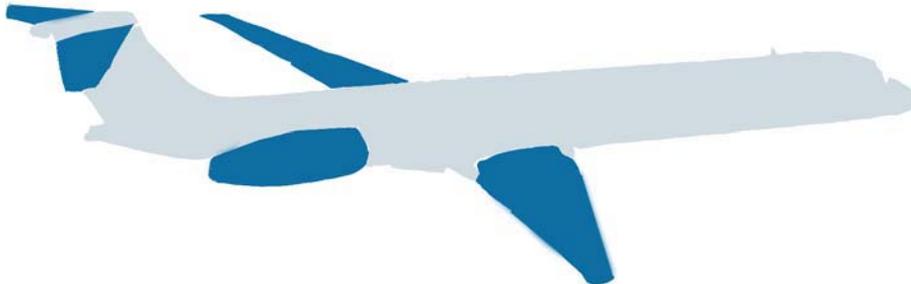
Planar Force Systems i.e. all forces act in one plane

Example: Airplane in straight, level flight - no acceleration. What are the forces required/available to keep it in this state?



Courtesy of NTSB.

Idealization (physical model)



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Include forces, dimensions, axis system, any other relevant information to allow us to calculate equilibrium of vehicle.

Write down equilibrium equations and solve for relevant forces/moments

Uses of Equilibrium

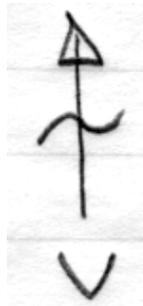
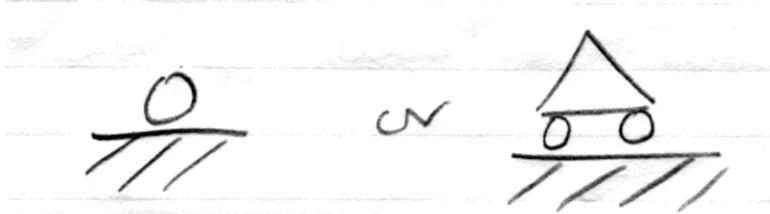
So far we have discussed external forces and the requirements for equilibrium. In most statics problems we can separate the external forces into applied forces and reactions due to the restraints or supports.

Types of support (and their reactions)

- Idealizations of reality (remember comments on modeling)

Common Idealizations

Roller

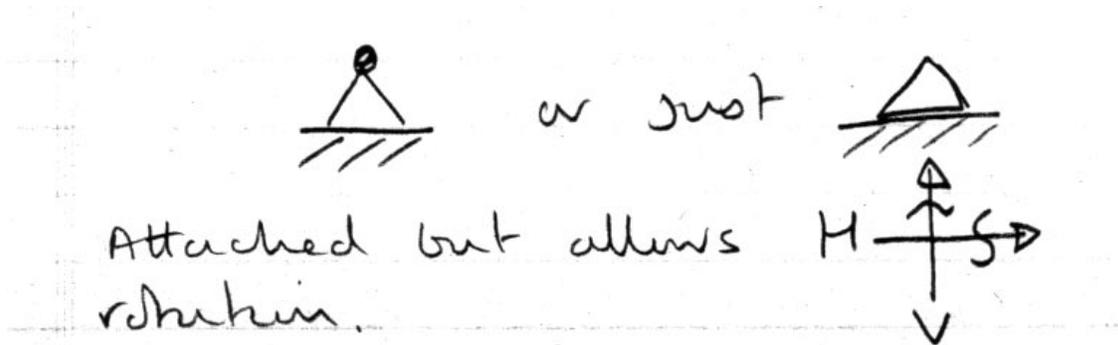


Squiggle through a force vector indicates a reaction.

Modelled as:

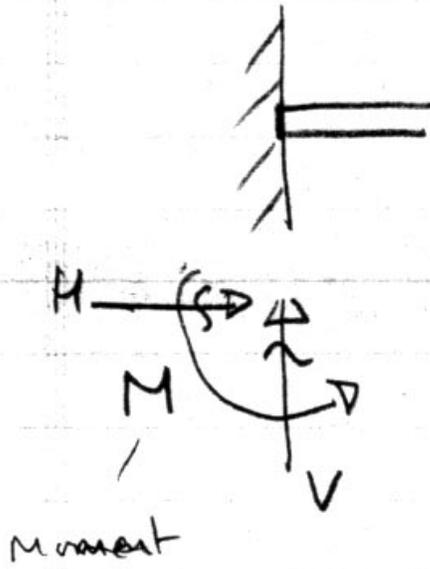
Reaction is perpendicular to support surface roller prevents any tangential reaction.

2. Pin (in - 2D a hinge, in 3-D a Universal joint)



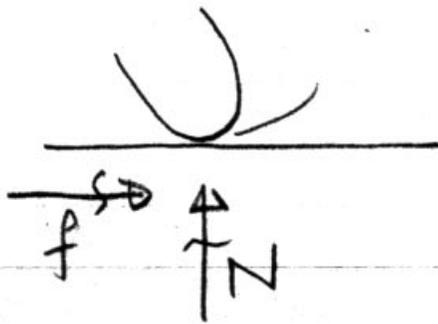
∴ Provides two components of reaction force, but no reaction moment, in arbitrary coordinate system.

3. Clamp



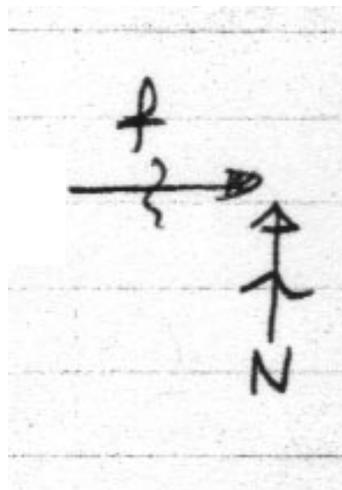
Structure is clamped, can impart two components of reaction forces, and one moment reaction (in 2-D) e.g., a rod built into in a wall.

4. Friction Surface



N = Normal force
 f = frictional force

} Modeled as:



Linked by

$$f \leq \mu N$$

where μ = coefficient of friction, f is parallel to plane of support

proportional to normal force, N

Acts opposite to motion that would occur if $f=0$

We can use these idealizations to separate the part of a structure that we want to analyze from the remainder of the structure. By replacing the real connections (supports) with the idealized supports, and then replacing the idealized supports with the equivalent reaction forces we can proceed with the calculation of the reaction forces via the equations of equilibrium. Usually we do this via drawing: A Free Body Diagram

Free Body Diagrams (FBD)

Allow isolation of structure/part of structure to be analyzed. Replaces external boundary conditions by equipollent force system

(similar idea to control volume/system, circuits etc.

There are two basic steps:

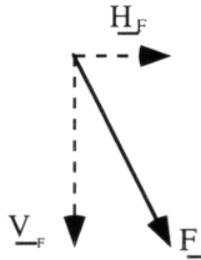
1. Make a neat diagram of the body with applied loads and idealized supports, important dimensions and coordinate system.
2. Draw a second diagram with idealized supports replaced by equipollent forces.

For Unified M&S a free body diagram is a good start to any structural analysis.

See Free Body Diagram Handout

FREE-BODY DIAGRAMS*Hints for drawing them*

- Draw an axis system
- Align axis system with structure or natural loading direction(s)
- Show important dimensions
- Show support reactions in positive orthogonal coordinates
- Label everything of importance
- Use solid lines for vectors (i.e., forces), dashed lines for their components



- Assume unknown forces (especially internal) are in tension. If they turn out to be negative (-) then they are in compression. Keep sense (i.e., tension or compression) you started with to avoid confusion.

(+) tension

(-) compression

- **BE NEAT!!!!!!!!!!!!!!!!!!!!!!**