

Lecture M2

So what Engineering Science methods do we use to do Structural Engineering?

We will use: Solid Mechanics (mainly Unified) and Structural Mechanics

Solid Mechanics is a branch of Newtonian Mechanics dealing with behavior of solids.

Mechanics: forces, masses and motion ---Review 8:01 in U lectures

Newton's Law:
$$F = \frac{d}{dt}(m\underline{v}) \quad (F = ma)$$

In Unified we will deal with structures in static equilibrium, $\Sigma F = 0$, but in general in aerospace engineering we often have to deal with structural dynamics (coupling of structural response and dynamics, particularly vibrations) and aeroelasticity (coupling of aerodynamic loading with structural response, also particular with regard to vibrations)

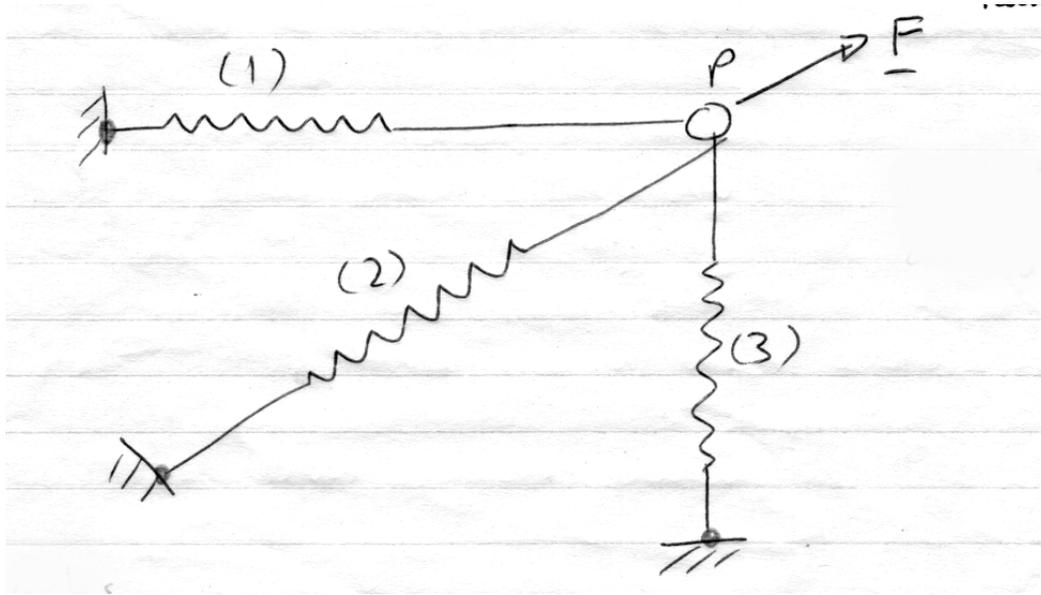
What is a solid?

During this term we will deal with:

"THREE (GREAT) PRINCIPLES OF SOLID MECHANICS"

Consider a body acted on by a force, restrained by three springs

(forces in spring = $k\delta$) Hooke's Law



3 Principles:

1. Equilibrium: Forces must balance
2. Compatibility of Displacements the springs remain connected - displacements at P must match.
3. Constitutive Relations (Force - deflection)

How much force is needed to cause a certain deflection (or vice versa).

e.g. spring $F = k\delta$

We will come back to all of these formally in the coming weeks.

"All About Equilibrium"

The concept of equilibrium is the first of the 3 great principles of solid mechanics we will consider.

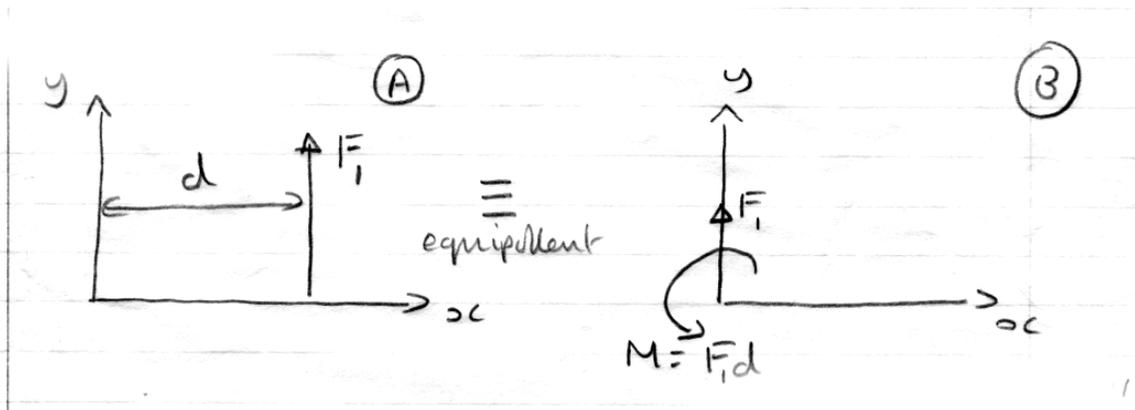
For a structure, we will need to consider the general case, but let's build up to this by considering the simplest case:

First, for formality, we will often "idealize" a relatively complicated force system as something simpler. This relies on the concept of "equipollence".

Equipollent Forces (equipollence means "equally powerful"),

Definition: two forces are equipollent if they have the same total force and total moment about the same (arbitrary) point.

Example: Two force systems, A and B, viewed with respect to the origin



Force F_1 at a distance d from the origin

Force F_1 at origin and moment M

A

B

$$F_A = F_1 \quad \Leftrightarrow \quad F_B = F_1$$

$$M_A = F_1 d \quad \Leftrightarrow \quad M_B = F_1 d$$

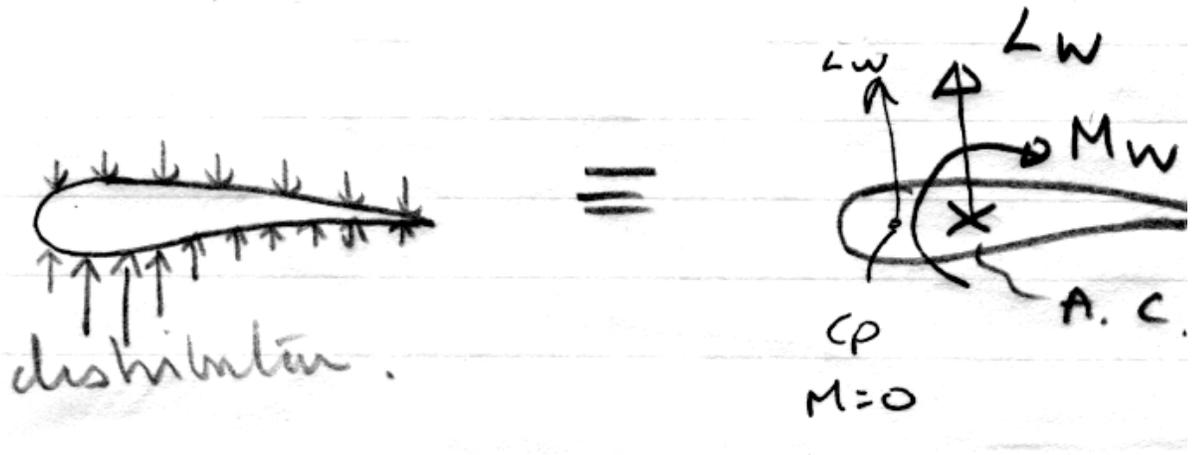
NOT Equilibrium ($\sum = 0$) but equipollence ($\sum = F, M$)

Generalize in 3-D

Must have $\sum F_i$ and $\sum M_i$ (external)

The same for both cases.

Examples of equipollence: Center of Gravity for a solid body
aerodynamic forces acting on a wing.



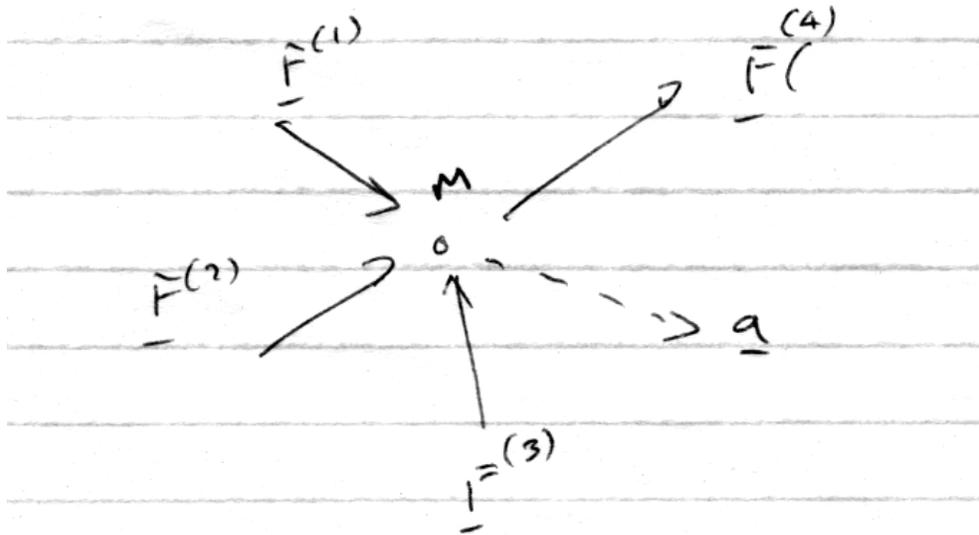
The actual distribution of pressure is given on the left. On the right we have reduced it to an equipollent set of forces. Either a lift force L_w acting through the center of pressure, or a lift force, L_w and a moment, M_w acting about the aerodynamic center.

Now we can move on to look at equilibrium itself, starting with the simplest case:

Equilibrium of a Particle

A particle is a body whose mass is concentrated at a point.

Forces acting on a particle produce acceleration.



Newton's 2nd Law gives:

$$\sum_i \underline{F}^{(i)} = M \underline{a}$$

In many cases in structures we have:

$\underline{a} = 0$ (statics) - note airplane in steady flight $\underline{a} = 0$, $v = \text{constant}$

So this reduces to (static) equilibrium

$$\sum \underline{F}^{(i)} = 0$$

Which represents 3 equations - vector addition

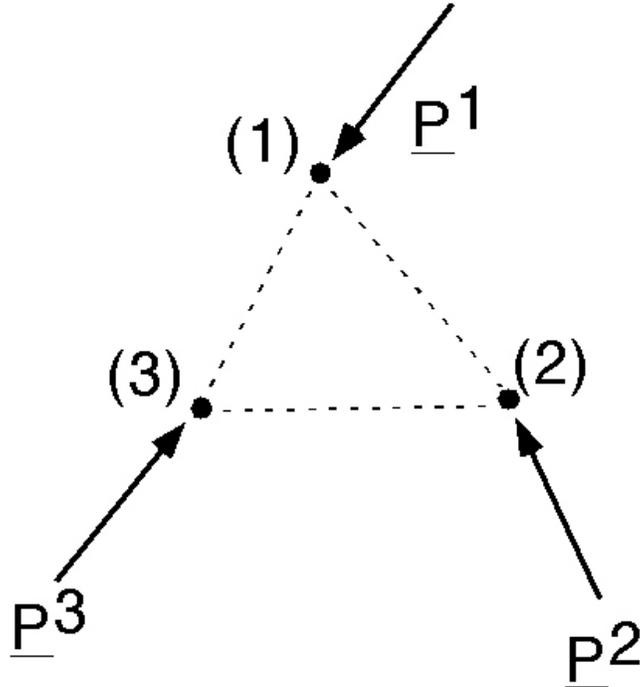
$$\sum_i F_x^{(i)} = 0 \quad \text{Sum of forces in } x = 0$$

$$\sum_i F_y^{(i)} = 0 \quad \text{Sum of forces in } y = 0$$

$$\sum_i F_z^{(i)} = 0 \quad \text{Sum of forces in } z = 0$$

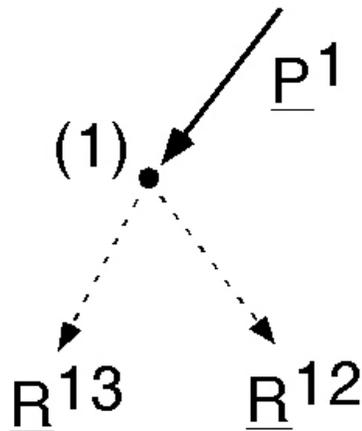
Equilibrium of a System of Particles

Look at a system of particles (we'll consider 3) with forces $\underline{P}^{(i)}$ acting on each particle i . Particles are connected by internal forces - exact nature of which does not matter at this time.



Isolate particle (1) - go back to equilibrium a particle

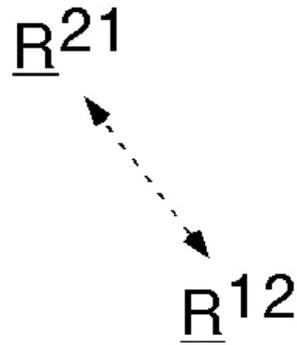
So for particle (1), draw a "free body diagram". (We'll discuss this more in later lectures.). Replace internal forces with equipollent forces, \underline{R}^{ij}



$\underline{R}^{(ij)}$ represents the reaction force in the connection between particles i and j .

Draw reactions initially in tension (i.e pulling away from particle)

Consider equilibrium of a connection between two particles



Newton's Law of action-reaction states:

$$\underline{R}^{(12)} = -\underline{R}^{(21)}$$

i.e. Point in opposite directions, but equal magnitude.

or more generally:

$$\underline{R}^{(ij)} = -\underline{R}^{(ji)}$$

Returning to the equilibrium of particle (1) in our system, we can now apply equilibrium

$$\underline{P}^1 + \underline{R}^{12} + \underline{R}^{13} = m_1 \underline{a}_1 = 0 \quad (1)$$

Similarly for particles 2 and 3

$$\underline{P}^2 + \underline{R}^{21} + \underline{R}^{23} = m_2 \underline{a}_2 = 0 \quad (2)$$

$$\underline{P}^3 + \underline{R}^{31} + \underline{R}^{32} = m_3 \underline{a}_3 = 0 \quad (3)$$

Combining these expressions gives (1), (2) + (3) and substituting for $\underline{R}^{(ij)} = -\underline{R}^{(ji)}$ gives:

$$\underline{P}^1 + \underline{P}^2 + \underline{P}^3 + \underline{R}^{12} - \underline{R}^{12} + \underline{R}^{23} - \underline{R}^{23} + \underline{R}^{13} - \underline{R}^{13} = 0$$

Hence:

$$\underline{P}^1 + \underline{P}^2 + \underline{P}^3 = 0$$

Which gives us the key result that internal forces do not appear in overall equilibrium of the forces acting on the system.

Is this enough to define the overall equilibrium of the system of particles?

We have dealt with the requirement of no linear acceleration.

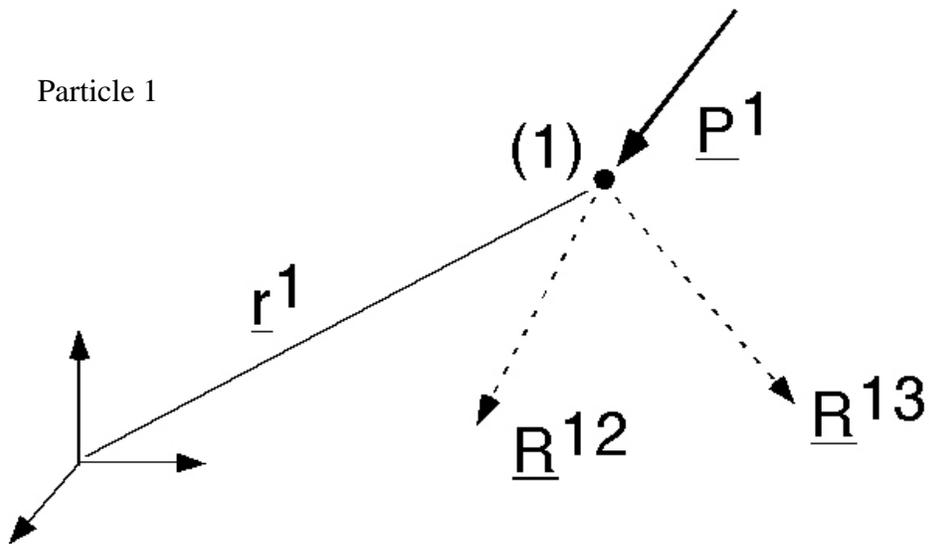
So there is a second set of equilibrium requirements.

Moment Equilibrium

For there to be no rotation, the moments about a (any) point must be zero.

(Why "any"? Pure moment magnitude does not change as point changes.)

Consider each particle and the moments acting on it. Take moments about some "convenient" point:



Moment due to a force: $\underline{r} \times \underline{F}$

Consider moment of forces acting on (1) about origin, O

$$\underline{r}^{(1)} \times (\underline{P}^{(1)} + \underline{R}^{(12)} + \underline{R}^{(13)}) = 0 \quad \text{M1}$$

Similarly for ② and ③

$$(2) \quad \underline{r}^{(2)} \times (\underline{P}^{(2)} + \underline{R}^{(21)} + \underline{R}^{(23)}) = 0 \quad \text{M2}$$

$$(3) \quad \underline{r}^{(3)} \times (\underline{P}^{(3)} + \underline{R}^{(31)} + \underline{R}^{(32)}) = 0 \quad \text{M3}$$

Again, use Newton's Law of action-reaction for the internal forces

$$\underline{R}^{(ij)} = -\underline{R}^{(ji)}$$

And we must further note that these pairs act along the same line of action, i.e., exert no net moment

Mathematically:

$$\underline{r}^{(1)} \times \underline{R}^{(13)} + \underline{r}^{(3)} \times \underline{R}^{(31)} = 0$$

Combining M1, M2, M3

$$\sum \underline{r}^{(n)} \times \underline{P}^{(n)} = 0 \quad (+ \text{inertial terms})$$

or

$$\sum \underline{M}^{(n)} = 0$$

i.e. only the external forces affect the overall equilibrium of the system of particles.