

**There were a lot of different matrices presented today. How can I keep them all straight.** I certainly don't expect you to memorize all of this. You should be aware that they all stem from the definitions of the engineering elastic constants – which are all defined in terms of simple loading (i.e one component of stress only). This means that the compliance form (known stress leading to resulting strain) is the most straightforward to follow directly, and I would expect you to be able to explain where this comes from, if not actually remember the terms in it.

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{zy} \\ \gamma_{zx} \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} \frac{1}{E} & \frac{-\nu}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ \frac{-\nu}{E} & \frac{-\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{zy} \\ \tau_{zx} \\ \tau_{xy} \end{pmatrix}$$

I do expect you to be aware of the fact that anisotropic materials exist and to have some physical understanding of why there are different elastic properties for a composite material in particular.

**Connection between tensor and engineering terms was confusing.** Fair enough. The most straightforward route to understanding this is by understanding what the terms in the compliance matrix mean (known stress to unknown strain – given by matrix in previous muddy response). These connect directly to the definitions of the Engineering elastic constants, and how they are defined. Once you understand this, it is a small step to convert the engineering notation stress and strain to tensor stress and strain.

**What exactly are  $\lambda$  and  $\mu$  in the 6x6 matrix?** They are just convenient groupings of E and  $\nu$ . All we have done is invert the 6x6 compliance matrix (see previous two questions) to allow us to go from known strain to unknown stress.

$$\mu = \frac{E}{2(1 + \nu)} = G$$

$$\lambda = \frac{\nu E}{(1 + \nu)(1 - 2\nu)}$$

**What exactly are  $\lambda$  and  $\mu$ ? Starting to understand tensors....** See previous response regarding  $\lambda$  and  $\mu$ ? I am glad that you are gaining confidence with tensors. They are not

really that frightening once you become familiar with them - they are just a convenient way of representing several simultaneous linear equations.

**What exactly are  $\lambda$  and  $\mu$ ?** See above.

**How are  $\lambda$  and  $\mu$  calculated?** See above

**What physically is  $G$ ?** It is the shear modulus. It links shear stress to shear strain, defined in engineering notation.

**I am wondering if the matrices make understanding the concepts any easier, because they have so many zeros.** I guess this depends on one's point of view. I agree that there are a lot of terms, and if you had to memorize them it might be confusing. However, the nice aspect to using a matrix presentation is that you can see quite readily how different terms link stress to strain in different directions – and that you can relate that back to your physical understanding of the engineering elastic constants.

**Order of initials:  $\nu_{LT}$ ,  $G_{LT}$  What do they mean?** The “initials” are better termed indices or subscripts. These are the Poisson's ratio and shear modulus in the LT plane of a transversely isotropic material, such as a unidirectional fiber composite. The Poisson's ratio,  $\nu_{LT}$ , is ratio of the strain in the transverse direction to the strain in the longitudinal direction for a load applied in the longitudinal direction

**Just wanted to clarify that we have learned three cases:**

- 1) **Uniaxial stress and strain**
- 2) **Isotropic material (not uniaxial) matrix**
- 3) **Transverse isotropic (which goes back to previous lectures) matrix.**

**Am I right or confused?**

We have certainly covered these topics. You need to distinguish the loading/deformation case (1) from the material response (2) and (3). The material response, as presented in matrix forms for linear elastic materials, is the constitutive law that links stress to strain and vice versa.

**Would have been handy to see appropriate (or both) matrices during first mud.** Not quite sure what you are asking here.

**Is whether a material is isotropic or not always determined by its internal composition, structure and independent of its external larger scale geometry?** Very good question. In defining material properties we are trying to separate out the material response from the structure in which it is used. For metals, polymers and ceramics this works very well, particularly for elastic materials. However, as we start to look at composites, we see that they are anisotropic because of their microstructure (fibers and matrix), if we were to make small structures from a composite material it might start to matter exactly where individual fibers were located, and a single set of material properties might not be sufficient. In general though, for the sorts of materials and structures we are considering, the material properties are determined by composition and microstructure at scales much smaller than those of the structures we are making from the materials.

**Also, the tensor notation for the matrices converting between stress and strain is kind of confusing. But engineering notation makes sense.** I am glad that you are confident with Engineering notation. Just having one set of notation straight in your mind is a good start. Beyond this it would be good if you understood simply that tensor notation and engineering notation are related, and in fact the only difference is the factor of two in the definition of shear strains.

**I was a little confused by the first PRS question because we had just used  $\mu$  for converting strain to stress and then it was used to represent  $10^6$ .** Thanks for the explanation, and I am sorry for the confusion associated with this.

**Do we need to know thermal expansion for the class or is it FYI only?** You should certainly be aware of the fact that materials can undergo deformation due to changes in temperature, and that we can handle it using the framework of elasticity. My main motivation for introducing it is to demonstrate that you can have strains without stresses and vice versa.

**Can you review thermal expansion for the fiber composite example?** I will mention it in tomorrow's lecture.

**Can you explain more of the actual meanings of the matrix operations we did for other situations like you did with the following: (axial loading of a block of a material in a rigid container) If you constrain  $\epsilon_{xy}$  then  $\sigma_{xy}$  does not equal zero by intuition.** Carl will cover this in recitation tomorrow.

**The last 10 minutes was too fast.** Yes, I agree, I will go back over this tomorrow.

**Went through last part (non isotropic) too fast. Would be good to see PRS on it to see if I actually know it?** Fair enough.

**Composite Strains.** See above.

**(Referring to the properties in the transverse plane of a composite – diagram with two squares at 45°) Why is the extension the same for both of these?** The extensions are not necessarily the same. The point I was making was that the elastic properties (Young's modulus, Shear modulus and Poisson's ratio) would be the same in any direction within the plane. Hence the term "transversely isotropic".

**Are we going to be going over the crystallography that was in the assigned reading?** I have lots of questions about that. Yes we will. In M22-25.

**Muddy: If you could just go back over relationships between elastic constants for anisotropic materials. I sure you are going to ....** Yes I will

**Mud – Why isn't  $\nu_{LT}$  one of the independent elastic constants?** It can be, but  $E_T \nu_{LT} = E_L \nu_{TL}$  so if three of these elastic constants are given values, then that fixes the fourth, so any three of them can be independent

**Are  $\sigma_x$  and  $\sigma_z$  automatically zero if the stress is only in the y-direction?** This is a very good point. We need to distinguish between applied loads and reactions. We may only apply a load in the y direction. But if the material is fixed, or attached to other material in the x and z directions, then this may result in reaction forces (stresses). So although we only load it in the y direction, unless we know that the x and z faces are stress free, then we cannot say that there will not be stresses in these directions.

**Will the strains in the LT and TT directions be identical for stresses applied in the longitudinal direction?** LT and TT are planes not directions. So LT is the plane containing the longitudinal fiber direction and one transverse direction. For loading in the longitudinal direction, the strain is isotropic in the transverse (TT) plane.

**Does  $\nu_{TT} = \nu_{LT}$  or  $\nu_{TL}$ ? If it doesn't what is the relationship between them? Or is there one?** No the transverse Poisson's ratio is not necessarily related to the LT or TL Poisson's ratios. It is in a different plane, and is governed by a different set of interaction between the fibers and matrix, so there is no reason why it should be related to the Poisson's ratios in the LT plane.

**On the Anisotropic viewgraph you used  $G_{TT}$  but this was not listed as a constant. Was this an error.** No, we will discuss in class tomorrow. It is linked to  $E_T$  and  $\nu_{TT}$ .

**In the second PRS question, is it because  $\epsilon_x$  and  $\epsilon_y=0$  that we know the solid is boxed?** Yes. A rigid container would not allow any transverse strains (not that there is any such thing as a perfectly rigid container of course).

**PRS questions (especially #1) were more calculations than concepts and were difficult to finish in 2 minutes.** Fair enough. As with all of these questions I am less concerned with whether you obtained the correct answer, than the thought processes you went through. In this particular case I was most intent on seeing whether you would automatically say that stress = strain \* Young's modulus – which is only true if the state of stress is uniaxial. So long as you were thinking that uniaxial strain implies multiaxial stress and vice versa I am very happy!

**Should we be working towards memorizing these matrices?** Absolutely not. You should have some idea where they come from and how to use them., but I will provide you with a formula sheet containing all the necessary information.

**Where did  $G = \frac{E}{2(1 + \nu)}$  come from again?** The fact that for an isotropic materials we can

look at a state of pure shear as being due to a biaxial tension/compression at  $45^\circ$  to the axis of pure shear. Given the isotropic material behavior, we would expect the two loading cases to result in the same deformation (strain state).

**Muddy points: I may have missed this, but can you please explain in words what  $\nu_{TT}$ ,  $\nu_{TL}$ ,  $\nu_{LT}$ ,  $G_{TT}$  etc. I think that I'm confused on what the subscripts represent physically.** There are definitely too many subscripts here, but it is inevitable when we have material directions in addition to the loading and deformation directions. I am using T to mean transverse, and

L to mean Longitudinal, with respect to a material direction, such as the direction of fibers in a composite. Thus  $G_{TL}$  is the shear modulus in the T-L plane.  $\nu_{TL}$  is the Poisson's ratio linking a strain in the L direction due to a load in the T direction.

**I don't understand the shear stress part of these matrices. What is G etc.** For an isotropic material G is the elastic constant which relates engineering shear strain,  $\gamma$  to shear stress,  $\tau$ . So  $\tau = G\gamma$

**Confused on the PRS question about transverse and longitudinal direction. Transverse and longitudinal with respect to what?** In the example for an orthogonal material as a laminated fiber-reinforced composite, the material directions are best described as longitudinal and transverse with respect to the fiber direction.

**And Chelsea on top of the premier league? Explain that. This defies all known laws of physics.** Chelsea cannot win the premier league - if they do the world will surely end!

6 cards with no mud, or positive comments. Thank you – clearly this was confusing lecture. I will spend some time clearing things up tomorrow.