

## Muddy Card Responses Lecture M11

**Clarify a bit more on tensors. A bit confused.** We will go over the aspects of tensors we need to use as we apply them to stress and strain tensors.

**In Einstein notation how do you tell the difference between  $F_{ij}$  and  $F_{ik}$  seems like both of these are  $F_{i1}+F_{i2}+F_{i3}$ , but there is obviously a difference (as in  $\sigma_{21} + \sigma_{23}$ )?** There is no functional difference,  $i, j, k$ , or for that matter  $p, q$  and  $r$  are just dummy symbols, that take values 1, 2 or 3. Remember the function of the indices is to indicate whether there are separate equations or summation across the indices, depending on whether the indices appear once or twice in a particular term.

**Last PRS question - confused?.** I will put this up again at the beginning of the next lecture. In the meantime you can look at the question and the solution at the bottom of these muddy responses.

**Please better explain the last PRS question (maybe another to make sure we got it).** See previous response.

**What is the stress tensor?** The stress tensor is the representation of the 9 possible components of stress acting on the faces of a cube orientated in a rectangular Cartesian coordinate system. i.e.

$$\sigma_{mn} = \begin{bmatrix} \sigma_{11} & \sigma_{21} & \sigma_{31} \\ \sigma_{12} & \sigma_{22} & \sigma_{32} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix}$$

You can think of this in terms of a matrix multiplication. The total force vector  $\underline{F}$  acting on the positive faces of the infinitesimal cube of material is given by:

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{21} & \sigma_{31} \\ \sigma_{12} & \sigma_{22} & \sigma_{32} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}$$

**Tensors are really confusing for what seems like an easy concept.** See previous response. This really is an easy concept, and is conceptually identical to matrix multiplication.

**Confused about what notation to use. See previous responses.** We will be using tensor notation for the rest of Unified for stress and strain, it will make things easier in due course.

I would like to have more practice with tensors. See previous responses

More info on how to use tensors; See previous response.

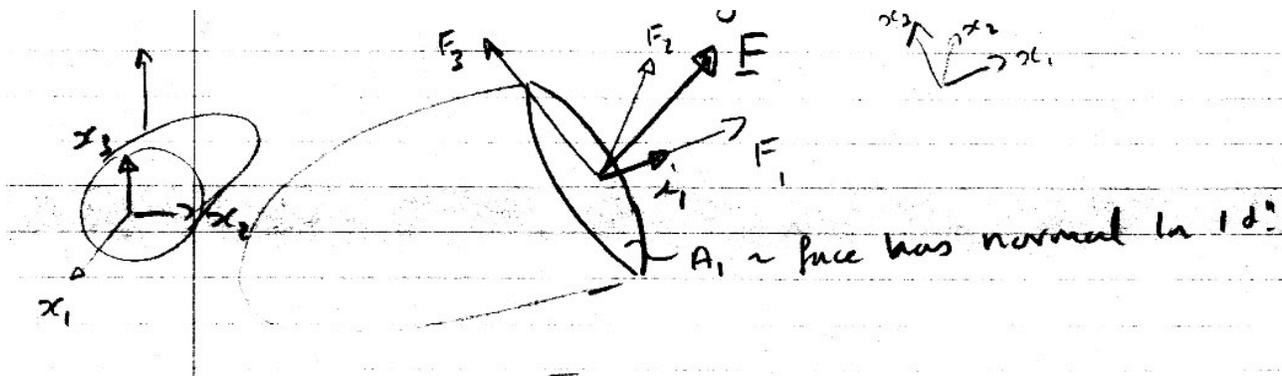
How do you get the balancing moments in the last PRS. We will look at this next time. Also, if you look at the notes for M11 you will see the answer.

Deck of cards. Are shear stresses on one face the same as normal stresses on another. We will talk about this in the next lecture. The key issue is that the cube of material must be in equilibrium.

According to constitutive behavior all materials deform linearly with temperature. Very good question. No, the constitutive law:  $\epsilon = \frac{FL}{AE} + \alpha \Delta T$  is just a model. Not all materials have a linear coefficient of thermal expansion and not all materials have a constant value of Young's modulus. That said, most of the materials we are interested in for aerospace applications do have nearly linear behavior, and so this is a useful model.

Does superposition mean you can break a problem into parts, solve the parts, then sum the answers? So long as the structure is governed by linear equations this is correct.

Can you review what is happening in:



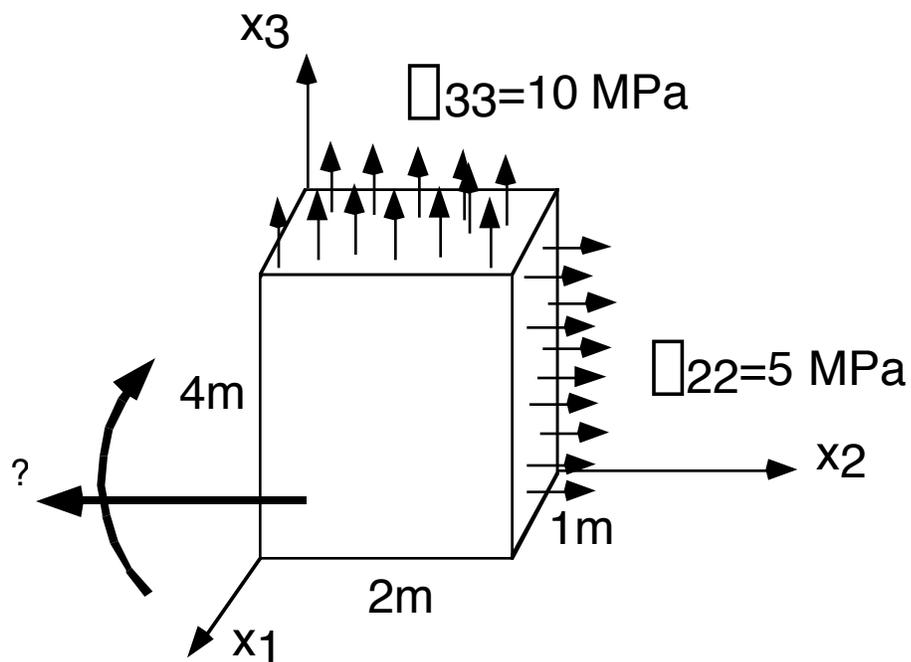
A force  $\underline{F}$  is being applied to a face  $A_1$ , such that the force is not aligned with the normal direction to the face,  $x_1$ . We divide the force into its components,  $F_1$ ,  $F_2$  and  $F_3$ .  $F_1$  is normal to the face and  $F_2$  and  $F_3$  lie in the plane of the face.

Aren't the stresses on opposite faces the same (drawing attached)? Yes and no. We will discuss this next time.

22 cards with no mud, or positive comments about the lecture. Thanks!

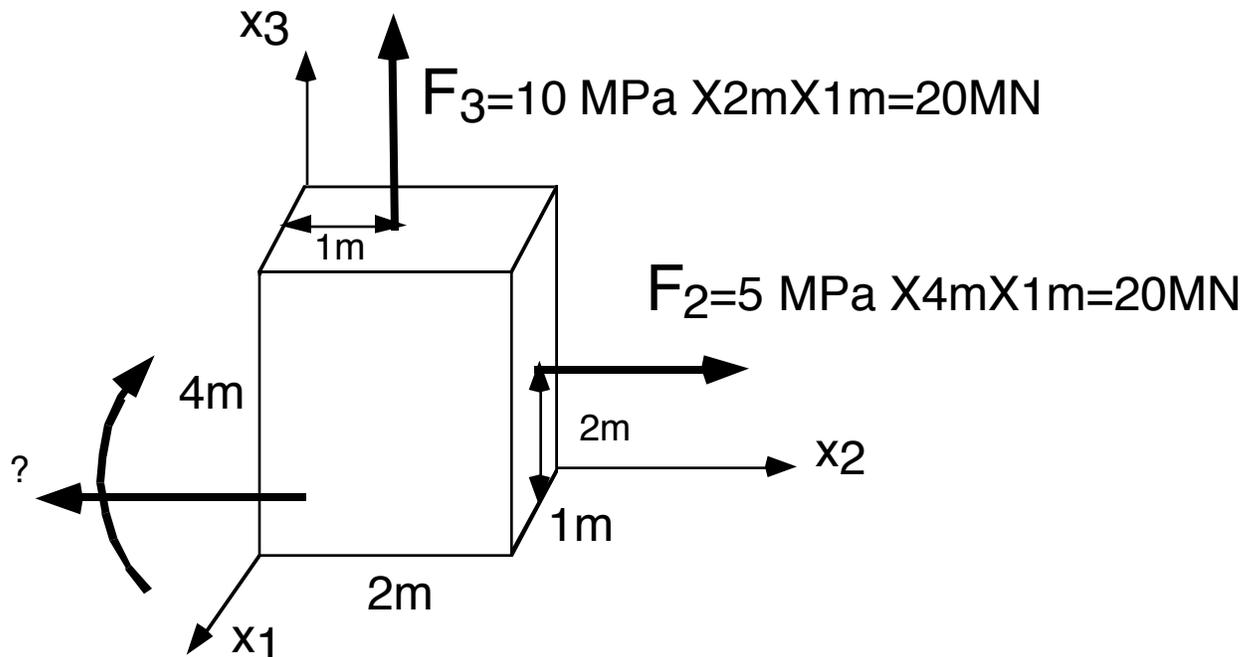
## M11 Concept Question 1

A block of material, sides 1m, 2m and 4m is loaded on two of its faces by uniform stresses of  $\sigma_{22}=5\text{MPa}$  and  $\sigma_{33}=10\text{MPa}$ . What force and moments need to be applied at the middle of the edge along the  $x_1$  axis (i.e. point  $(0.5,0,0)$ ) to keep the block in equilibrium



## M11 CQ1 Solution

Calculate equipollent forces and moment due to stresses. Multiply stress by area to obtain force acting at center of area (uniform stress) and then by moment arm to obtain moment.



Equipollent moment =  $20\text{MN} \times 1\text{m} - 20\text{MN} \times 2\text{m} = -20\text{MNm}$

Acting about  $x_1$  axis. Therefore solution is:

1. A force of  $(0, -20, -20)$  MN and a moment of  $(+20, 0, 0)$  MNm