

Muddy Card Responses Lecture M6 2/12/2004

Synopsis. Continued derivation of equations of simple beam theory. Obtained three key equations: Moment-curvature, Moment-Stress and shear force-shear stress. Also introduced second moment of area as the property of the beam cross-section that defines the stiffness of the beam, in conjunction with the Young's modulus of the material.

You went way to fast on the last part. I know, but we will review more slowly tomorrow.

Last part of the lecture a bit confusing. What is Q? Q is the first moment of area of that part of the cross-section above the plane defined by z , the location at which we are evaluating the shear stress. We will review tomorrow.

Not quite keen on what "Q" is. Fine, we will discuss further.

I have no idea what actually happened for the last five minutes of lecture. OK, see above.

Real question: We've only been talking about pure moments on beams, what happens when they're loaded elsewhere? Good question. We will discuss this tomorrow.

More explanation of the shear force/stress stuff at the end of lecture. Example would be very helpful. Will do.

Can you review the last part of the lecture tomorrow. It hit me a little quickly and I didn't understand. Will do.

Moved to fast, even through the first parts. Thankyou for the feedback. We will recover this material over the next lecture or two – I hope that this will consolidate things for you.

A bit too fast. Understood.

Beams must have a large length relative to their thickness to be considered as a beam, but when we analyze individual cross-sections the dimensional relationship is switched. How does this affect the assumptions we make during analysis? The assumption of slenderness allows us to make the assumption of plane-sections remaining plane, from which everything else flows. We will examine non-slender structures in the next couple of lectures.

If we vary the cross-section of a beam along z , and the stress σ_{xx} remains constant, then does the beam still bend in a circular arc? It doesn't make sense logically that it would, but if there is some way to explain it does, please do. Thanks. This is a perceptive question. The beam no longer has the same cross-section, and therefore although the moment is the same at any section the stiffness of the beam is not. The curvature of the beam will therefore vary with position along the beam and so we will not obtain a circular arc. We can still integrate up the moment-curvature relationship to obtain the deformed shape of the beam.

I find it counter-intuitive that a sloping beam (varying cross-section with x) can be analyzed in the same way we did for the uniform one. If you apply similar moments at the two ends will it still form part of a circular arc? No – see explanation above.

Mud: In PRS #1 why doesn't the shape of the object affect the stress equation. Because

$\sigma = \frac{Mz}{I}$ applies at any given cross-section. If I is a function of x , then that is OK, the equation will take care of it. Remember this stems from plane sections remaining plane. Any and every cross-section will remain plane so this equation applies everywhere, so long as the beam is made of an homogenous material (no layers of dissimilar materials).

Could you please review the reasoning behind the concept question?

Are you saying that the stress is not linear through the thickness of the beam because your equation (4) $\sigma_{xx} = \frac{\sigma_{xx}}{E_x}$ is based upon the assumption that the strain and stress both vary

linearly? You mentioned strain is linear though stress is not, this was confusing. You are on the right track. See explanation above. The strain varies linearly, consistent with plane sections remaining planar and perpendicular to the neutral axis of the beam. However, if there are different materials through the cross-section, the different materials experiencing the same strains will experience different stresses – so a linear strain distribution will not produce a linear stress distribution.

Can you explain conceptual differences between I and Q again? Physical meanings. In future please give me something to work with here – what do you understand? I is a property of the whole cross-section. It reflects how material is distributed away from the centroid of the cross-section. A higher value of I will produce a stiffer beam. I is termed the second moment of area. Q is a special case of the first moment of area. It is the first moment of area (integral of z , not z^2) for that part of the cross-section more than a distance “ z ” from the centroid of the cross-section. It allows us to calculate the shear stress at that z location.

Still don't understand negative signs for F, S and M integral equations. There is no negative sign in F . The negative signs in the expressions for S and M arise due to the sign conventions for positive shear stresses and tensile stresses acting in the opposite direction to the sign convention for positive S and M . **Don't understand PRS 1.** See above. **Lost last 10 minutes.** I will attempt to recover next lecture.

In beams $F=0$, why is there σ_{xx} ? Where is the force for the xx stress coming from? The moment? This is absolutely correct. You could think of the bending moment on a particular cross-section as a couple made up of a net compressive force on the top half of the cross-section and a net tensile force on the lower half of the cross-section. The two forces are equal in magnitude and opposite in direction. Thus, they have no net force, but because they are off-set from each other, they do exert a moment.

Is I_{yy} the same moment as that in $\sigma_{xx} = \frac{Mz}{I}$. Thinking yes. Yes, you are correct, I_{yy} is the second moment of area about the y axis (i.e. the one we are interested in). It should technically be written as I_{yy} since it is a second order tensor quantity. You will meet this if you take 16.20. Since we are only considering bending about one axis in Unified I will often just use "I".

Is there any reason why you refer to I as the second moment of area? My roommate the civil engineer likes to talk about just "the second moment" Having often heard tell about this mythical second moment, I was just wondering is this it? Presumably yes. I would encourage you to avoid calling I simply a moment – since it should not be confused with a moment due to a force acting with a moment arm (i.e $P \times L = M$), also confusion with a dynamic moment of inertia can become confusing if you ever worry about the structural dynamics of structure.

So is the purpose of the equation $\tau_{xz}(z) = \frac{SQ}{I_{yy}b}$ to show that a positive shear force exerts a negative shear stress? What do the subscripts I_{yy} represent? The key point about $\tau_{xz}(z) = \frac{SQ}{I_{yy}b}$ is that it links the shear force to the shear stress on the cross-section of the beam. The minus sign just arises from the conventions for positive shear stresses and shear forces. For an explanation of I_{yy} , see previous explanation (two above).

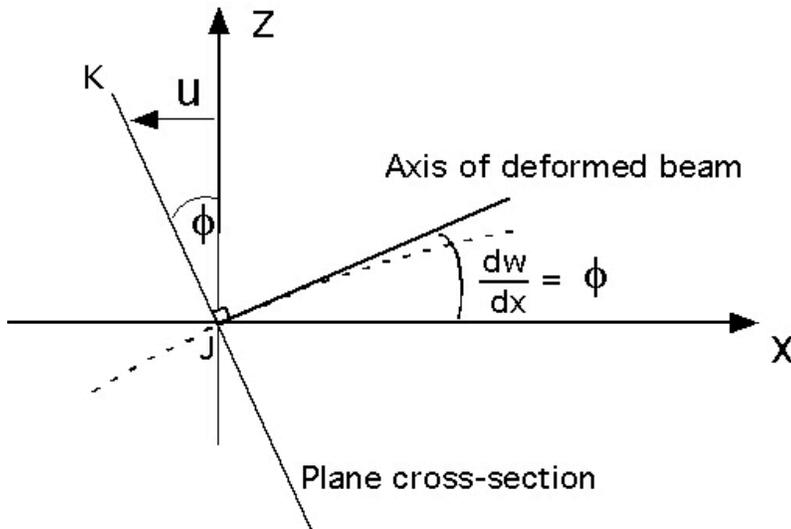
Could you explain again why choices A and C on the PRS question do not alter the linear moment-stress relationship. This is because the plane-section remains plane assumption results in a linear distribution of strain on a given cross-section. If the cross-section is made of the same material throughout then a linear variation of strain implies a linear variation in stress. Hence $\sigma_{xx} = \frac{Mz}{I}$ works. Neither A nor C introduced anything to violate the plane-sections remain plane and perpendicular assumption.

How do you know strain in a beam made of several materials varies linearly with position along cross-section? Does this all come back to the "plane sections remain planar" concept from the beginning? Exactly right. Remember we had made no assumption at that point regarding the composition (materials) of the beam. In going from displacements to strains, we were still completely general regarding cross sectional shape and materials. It was only when we converted stress to strain (via our elastic constitutive law) that the material distribution mattered.

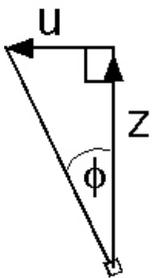
And a random question... Do you know anyone taking Chinese brush painting through MIT's student art association? - yes my wife!

What would happen if M weren't constant? Are we going to see that in Unified? Absolutely – we will examine in lecture M7.

Still don't understand what u is and why we want to find it. u is the axial displacement of a point on the cross-section of the beam in the x direction. Similarly w is the displacement in the z direction. We want to find u since this leads to the extensional strain and hence to the tensile stress distribution.



Perhaps a clearer picture of what we are talking about would have been:



There were 7 muddy cards with no mud, or positive responses.

About 1/5 of drum majors carry a mace and most do not twirl it. You were probably thinking of a baton twirler during your demonstration. Thank you for the explanation – I confess great ignorance of these matters.