

Synopsis. Continued development of model for buckling of columns. General solution of

form:  $w = A \sin \sqrt{\frac{P}{EI}}x + B \cos \sqrt{\frac{P}{EI}}x + Cx + D$  where A, B, C, D are determined by

boundary conditions. In general solutions for the buckling load are of the form:  $P = \frac{c\pi^2 EI}{L^2}$

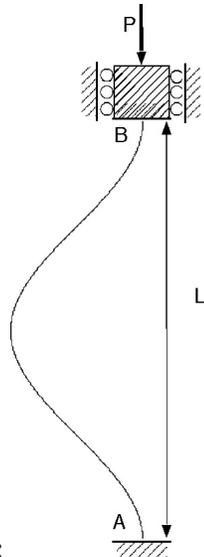
where "c" takes a numerical value depending on the boundary conditions. Discussed effect of imperfections due to initial curvature or non-axial loading. Imperfections lead to the  $w(x)$  being continuously defined as a function of load, therefore removing the bifurcation behavior. The ideal buckling load remains as an asymptote to the p-w response.

**I understand that the buckling state has less energy involved than axial deformation after a certain load P. Is there any other type of deformation (what kind) that involves less energy that mode 1 buckling (beside the beam breaking or permanent bending position)?** Not within the assumption of our model. This reminds me that I should have pointed out that buckling will occur in the transverse direction with the lowest "EI" value. I will mention this at the beginning of M15 on Monday.

**Is L in the buckling examples equal to the original length of the beam before applying loads? Or is it the length after applying loads?** Remember we are making the assumption throughout that the deflections are small, so in terms of  $x=L$  assumptions on the boundary conditions L is unchanged during deformation. If this difference is large enough that it starts to matter in the equilibrium equations then we will need to use a large deformation model for our analysis.

**Recently back home 12 oz coke cans have been replaced by 8 oz coke cans. (L = 2/3L original) Would you expect the load at which P is at the bifurcation to go up or down?** In the axial buckling of cylinders L is not so important as the ratio of the wall thickness to the radius (t/R) as this ratio goes down, so will the buckling load. I suggest that you do the experiment!!

**With the application of the load at a vertical distance e from the ends of the beam, I don't understand how the load is distributed axially. Is it still uniform? Why? Why is the only difference the moment at each end?** Think about the equipollent force case (lecture M1 or M2 last term). A load applied at a perpendicular distance d from a reference point can be replaced by the same load P and a moment Pd at the reference point to create an equipollent force system. This is all we have done here.



**Mud:** if you were to solve: **would you include the reaction forces at the imaginary pin rollers?** **Guess:** I am thinking no, that you would just use the same boundary conditions as pin rollers at the inflection points. I did not do a very well at explaining this. I should have worked through a solution via:

$w = A \sin \sqrt{\frac{P}{EI}}x + B \cos \sqrt{\frac{P}{EI}}x + Cx + D$  and invoke the boundary conditions of zero slope and zero transverse deflection at  $x=0$  and  $x=L$ . If you do this you will find that the critical buckling load is given by  $P = \frac{4\pi^2 EI}{L^2}$ . The observation of there being zero moment at the  $1/4$  and  $3/4$  points was meant to help you develop some physical intuition regarding this sort of problem, but I think that it failed!

**Is the mode=3 for the example you gave (see previous question). If it does, does the constant become**

. No. It is not being pushed into the third mode. You

could look at this as equivalent to the second mode, so  $c=4$ . (i.e.  $n^2=4$ ), however the boundary conditions have changed from the simply supported case.

Why don't the extra bits of column contribute to the critical load? (i.e. for  $x < L/4$  or  $x > 3L/4$ ) ? In a structure with multiple sections it is the section with the lowest buckling load that determines the critical buckling load for the whole structure. So the  $L/4$  lengths at either end will have a higher buckling load than the  $L/2$  length between the two inflection points.

**Is this structure possible in real life?** I.e a friction less piston perhaps. That is one example. In fact it is not a bad representation of a steel column in a multi-story building between rigid connections where floors are attached.

**I am badly confused on the "c" issue. If you could explain it again that would help.** OK. All I would like you to take away from this discussion is that different boundary conditions lead to different values of the buckling load. However, they are always of the form

$P = \frac{c\pi^2 EI}{L^2}$ . Stiffer boundary conditions (more restraining) will lead to higher values of  $c$ .

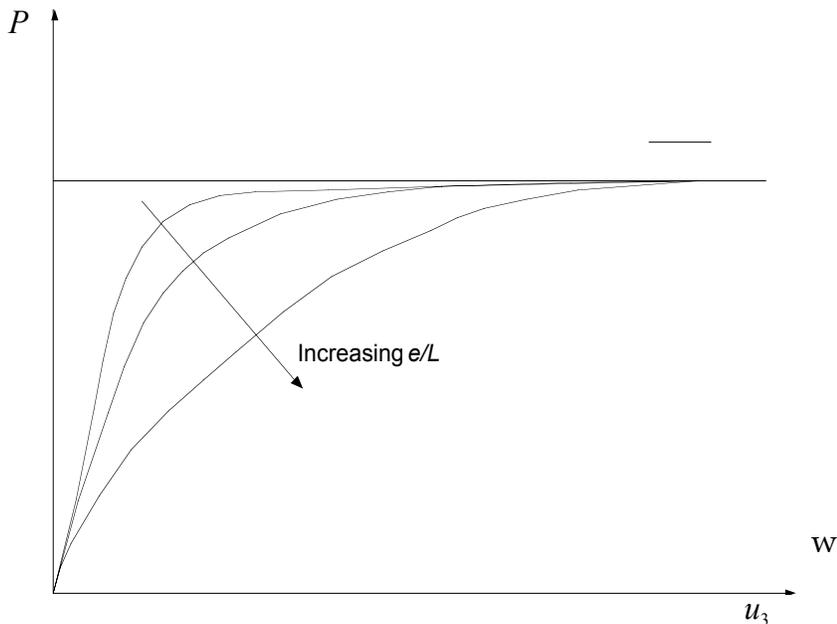
Another way to think about this is to consider  $c$  as a multiplier on the effective length of the column.

**Very confused about the first PRS question. How did you calculate “ $c$ ”.** See above.

**How do you find the  $c$  constant in the  $P_{crit}$  equation?** See above. **How does this compare with the  $n^2$  term found from the differential equation last time?** They are different, but related, concepts. The  $n^2$  term represents the higher buckling modes on a beam with a particular set of boundary conditions. Different values of “ $c$ ” correspond to different boundary conditions.

**Would the column pinned at one end still be equivalent to one twice as long pinned at both ends if it weren't of negligible mass?** It is not so much the mass as the weight that matters here. The weight of the column will act as a distributed dead load. If this is a significant fraction of the applied load then it will need to be taken into account. This will result in a different governing differential equation for buckling, which will include a term for the dead weight, which will vary with  $x$  and the applied load, which does not.

**I don't understand what you are showing on this graph:**



It is plotting the axial applied load vs. the resulting transverse displacement (of a representative point on the column – say the mid point).  $e/L$  is representative of the

magnitude of the imperfection in the loading (i.e. how non-axial the loading is). With increasing eccentricity to the loading we move away from the ideal bifurcation behavior.

**Does bifurcation occur at the critical P? What does it mean physically?** Yes it does occur at the critical load. Physically this means that if one had a perfectly straight column, loaded perfectly axially, then at that critical load, if there was any slight perturbation in the load (or straightness of the column) then the column would “snap” into the buckled state. Much like the coke can that Meredith was standing on.

**Why is there no bifurcation behavior if A, B, C, D are defined?** If these values are defined then  $w$  is a continuous function of  $P$ . This means that there is no sharp bifurcation point in the  $P$ - $w$  response.

There were 5 muddy cards with no mud, or positive responses. I wonder if I always receive the same 5 muddy cards? I also wonder if the 40+ people in the lecture who don't fill out muddy cards are mud free or are so profoundly confused that they don't know where to start writing a muddy card – it is questions like this that cause professors to remain sleepless at night. Any thoughts would be most welcome.