

Muddy Card Responses Lecture M11 2/21/2004

Synopsis. Completed model for circular shafts. Discussed limitations of model for non-circular cross-sections. Discussed analysis of distributed torques. Moved on to begin to discuss elastic instabilities. Considered collapse of a rigid rod, with a torsional hinge at one end. Found that combined axial and transverse loading could lead to an effective stiffness less than zero, corresponding to collapse of the structure. In the “ideal” case of a purely axial load, “bifurcation” behavior would result.

What happens when P_2 is greater than K_t/L for the scenario given in class today?. The rod is not stable under a load P_1 and so collapse occurs. i.e θ grows in an unbounded manner.

What is the source of sounds when you buckle the composite carbon fiber? The composite plat is moving fast enough to generate a compression wave (a sound wave) in the air with a high enough frequency and large enough magnitude that we can hear it.

A bit confused with the last example. Not clear what the effective stiffness is. If we rewrite the equilibrium equation as: $P_1 = \left(\frac{K_t - P_2 L}{L} \right) \theta$ then $\left(\frac{K_t - P_2 L}{L} \right)$ appears as a constant of proportionality between P_1 and θ . By analogy to the familiar $P=k\delta$ for a spring, $\left(\frac{K_t - P_2 L}{L} \right)$ is the “effective” stiffness. It is not actually the stiffness because it depends on the load P_2 that is also being applied.

Last PRS question was not clear. Not sure about how the stiffness goes to zero. See answer above. I will spend some more time on this in M12.

How do you determine the certain θ when a beam will buckle? Actually there is no given θ , it is purely a function of load. Once buckling occurs, θ , is undefined. Buckling does occur at a given value of the axial load, P_2 .

If there were no imperfections/deformations in a beam could you in theory apply an infinitely large P_2 ? Is this what the bifurcation point means? Of course in real life even if you could get a flawless structure small imperfections would form as you increased the loading. This is exactly correct. There is nothing I can add to this.

Assuming that you can apply a load perfectly straight at the bifurcation point, which way would the beam buckle? See mud above. Mathematically you cannot tell which way it will go. Physically there is no such thing as a perfect column or a perfect axial load.

Thinking about buckling you design for buckling in a certain direction with the top of a jab and the safety seal on it that pops when you open it, right? You are absolutely correct. Good example.

Question: the buckling state a state of second equilibrium? Answer: it seems so by watching the ruler bend. It seems to bend only to a certain point. Good observation skills. There are actually two things going on here. Firstly if I was loading it with a dead load (constant force) then it would actually collapse. In fact I was loading it more in displacement control, in which case the load drops as the stiffness drops and the structure does not collapse. Secondly, as the deformations become large the structure becomes stiffer again – so this resists the buckling deformation.

In PRS 2 if $P_2 \geq K_t / L$ and for $P_1 \approx 0$ how is $\theta = \infty$? Just work through the equilibrium equation, rearrange so that you have: $\theta = \frac{P_1}{\left(\frac{k_t}{L} - P_2\right)}$ and this result becomes clear.

F=Kx, why is a moment the same. I am not quite sure what you mean by this question. One can make a torsional spring such that $M=k_t\theta$, obviously the units of k_t are different from $F=kx$, due to dimensional considerations.

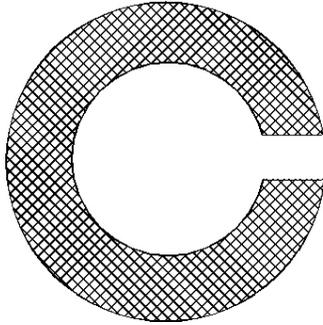
Silly question, but why does the spring in the last example apply a pure moment instead of a force in the $-\theta$ direction? Silly answer – because I chose it to act that way!. Serious answer: torsional springs do exist in practice – watch springs being the obvious example. It is possible to generate a pure moment without a net force – so this is physically reasonable.

I thought triangles were the strongest geometric structures, but in lecture you said a hollow cylinder is best. When is a triangle the best selection?. Triangles are never a good cross-section for beams or shafts. Where triangles give strength and stiffness is in the layout of bars in a truss. If a truss is not made up of triangles then the bays of the truss will be mechanisms.

After buckling a member can still support a reduced load? How do we know what that is? Post buckling behavior (as it is called) is beyond the scope of Unified. It requires knowledge of the material properties and large deformation theory.

If you want to start with “last lecture” notes, could you please start 5 minutes past the hour. I am behind almost every lecture because I am struggling to write down the “last lecture” notes. It seems only fair since that is when class starts. ? My apologies. I perhaps should have made it clear that I really do not expect anyone to copy down these notes. They are intended as a 5 minute summary of what was said in the previous class. They are intended to help you get up to speed with the topic at hand, ask any questions hanging over from the previous lecture, and a chance for me to deal with any muddy comments that I feel need to be expanded on. I generally do not put any new material up in these notes – only material that is in the notes from the previous lecture. I am sorry for the stress that this is obviously causing, I sincerely encourage you to relax, just listen to what I am saying, and make sure that you are following along.

Why hollow circular section over a hollow square section. Because it will have a higher second polar moment of area for a given cross-sectional area.?



Why doesn't: work? This is because at the cut in the circular section there are free surfaces which cannot support a shear stress. The shear stress builds up over some circumferential distance away from the cut. As a result the total torque that the cross-section can carry is drastically lower than the closed circular cross-section.

In the shaft with the slit in it does the integral $J = \iint (r^2) dA$ reflect the extent to which stiffness is lost? Or are there modelling assumptions that break down in that situation.

Good question, good thoughts on the response. The modeling assumption of plane-sections rotating as rigid bodies do breaks down for this case. The initially rigid cross-section will twist. I will do the example at the beginning of lecture tomorrow.

Last PRS question solution B is unclear. This was the statement that equilibrium no longer applies. This is because once the bar collapses it becomes a dynamic problem and we need to solve Newton's equation of $F=ma$ rather than static equilibrium.

T, Tr, τ what's the difference. T is the torque with units of N-m, τ is the shear stress with units of N/m^2 , and r is the radial position, with units of m.

There were 5 muddy cards with no mud, or positive responses. And one suggestion for a useful lab experiment – which we may do in the next couple of lectures. Thank you.