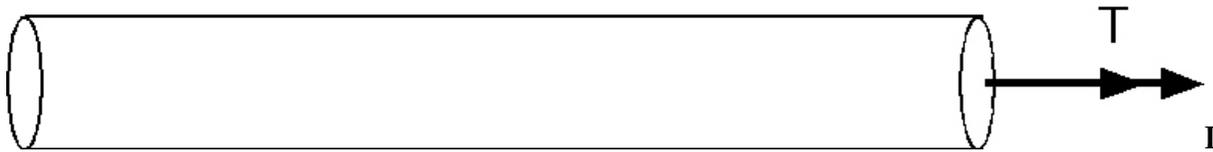


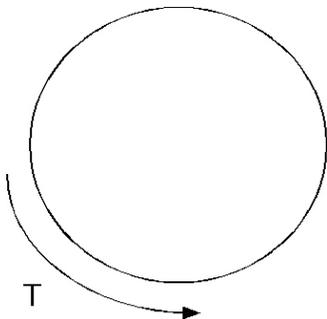
Synopsis. Developed the model for torsion of circular cross-sectional shafts. Took assumption of deformation of a cross-section consisting of a rigid body rotation about the long axis of the shaft through a twist angle, ϕ . Obtained expressions for strains from displacements of arbitrary point on the cross-section. From strains we obtained stresses via the shear modulus. We related the stresses back to the equipollent torque acting on the cross-section. From this obtained the torque-twist and torque-shear stress equations.

What is the equation for the torque? Same as moment? In the context of slender structural members a torque is a moment which acts about the long axis of the slender member, a bending moment acts about a transverse axis.

On board you drew:



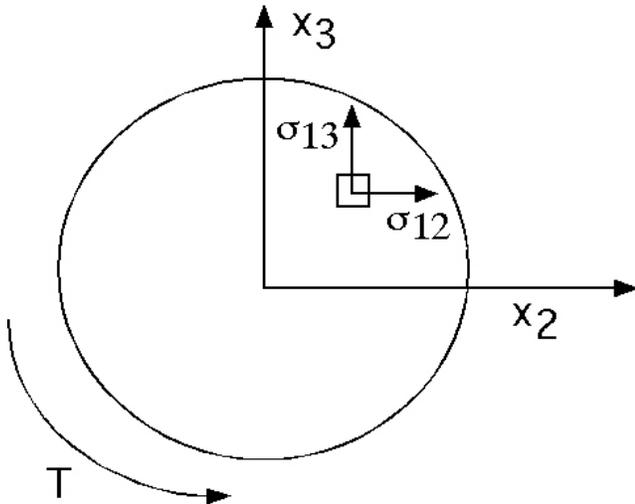
thought we were just dealing with



The double arrow sign in the first figure indicates that T is a torque in vector form. I should have written it as \underline{T} . The two cases are equivalent. The second figure (b) indicates a torque acting about the x_1 axis (coming out of the page at you).

Don't understand sign conventions for torque-double integral relationship, equation 8

$$\text{equipollent torque, } T = \iint (x_2 \tau_{13} - x_3 \tau_{12}) dx_2 dx_3 \quad (8)$$



τ_{12} and τ_{13} are positive shear stresses according to our definition of shear stresses. The x_1 direction is coming out of the page (screen) towards you, so τ_{12} and τ_{13} are acting on a positive x_1 face in positive x_2 and x_3 directions. T is a positive moment, as defined by the right hand rule, in vector form it acts parallel to the x_1 direction. With these definitions, τ_{12} makes a negative contribution to T and τ_{13} makes a positive contribution. This is all self-consistent.

When we worked with beams we analyzed beams that had multiple point loads along the x_1 axis. How do we analyze circular shafts that have multiple point torques applied along the x_1 axis? We will discuss this on Monday.

Will we ever get to analyze shafts with axial as well as torsional stresses? This case certainly physically exists – think about how a drill bit works (there is typically a torque and an axial compressive load). We will not develop a model for this in Unified though. 16.20 will cover this general loading if you decide to take that class.

Is there an easy way to turn the strain equations into polar coordinates? Since $dx_2, dx_3 = r dr d\theta$ I was thinking about some sort of process using that at radius. It is certainly possible to represent the equations of elasticity in polar coordinates (or any other coordinate system for that matter). A quick search of the web led me to:

<http://www.maths.man.ac.uk/~mheil/Lectures/Elasticity/chapter5.pdf>. If you are interested you should take a look.

How useful is the small deformation shaft model – wouldn't an application (like an engine shaft) undergo large deformations? How much will we actually see small deformation shafts in aero? Good question. The model is very good. Unfortunately you will not actually get a chance to compare it with experiment, but it works very well for circular cross-section shafts. We generally want drive shafts to behave in the small

deformation regime. Think what would happen if a shaft went through a large angle of twist in transferring a torque from a turbine to a compressor or an engine to a propeller. Firstly there would be a non-linear variation in the torque transmitted with the torque applied, which would be difficult to control, and would ultimately limit the amount of torque that could be transmitted. Secondly, an overly flexible shaft leads to problems in the dynamics of the shaft. Torsional and bending vibrations of shafts are major design constraints on how slender and flexible a shaft can be made. We therefore always tend to keep shafts operating in a small strain/ deflection regime.

There were 15 muddy cards with no mud, or positive responses. Thank you.