KEY CONCEPTS FOR MATERIALS AND STRUCTURES

Handout for Spring Term Quizzes

Basic modeling process for 1-D structural members

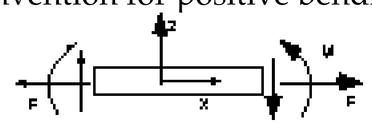
(1) Idealize/model – make assumptions on geometry, load/stress and deformations
(2) Apply governing equations (e.g. equations of elasticity)
(3) Invoke known boundary conditions to derive constitutive relations for structure (load-deformation, load-internal stress etc.)

Analytical process for 1-D structural members

(1) Idealize/model – assumptions on geometry, load/stress and deformations
(2) Draw free body diagram
(3) Apply method of sections to obtain internal force/moment resultants
(4) Apply structural constitutive relations to relate force/moment resultants to
   a) internal stresses
   b) deformations (usually requires integration – invoking boundary conditions)

Elastic bending formulae

Based on convention for positive bending moments and shear forces:



for continuous loading, q, $q = \frac{dS}{dx}$, $S = \frac{dM}{dx}$

Bending of a symmetric cross section about its neutral axis (mid plane for a cross-section with two orthogonal axes of symmetry).

$$\sigma_{xx} = -\frac{Mz}{I} \qquad M = EI\frac{d^2w}{dx^2} \qquad \sigma_{xz} = -\frac{SQ}{Ib}$$

where σ_{xx} is the axial (bending) stress, M is the bending moment at a particular cross-section, I is the second moment of area about the neutral axis, z is the distance from the neutral axis, E is the Young's modulus of the material, w is the deflection, x is the axial coordinate along the beam, σ_{xz} is the shear stress at a distance z above the neutral axis, S is the shear force at a particular cross, section, Q is the first moment of area of the cross-section from z to the outer ligament, b is the width of the beam at a height b above the neutral axis.

Second moment of area $I = \int_A z^2 dA$

Standard solutions:

Rectangular area, breadth b, depth h: $I = \frac{bh^3}{12}$ Solid circular cross-section, radius R: $I = \frac{\pi R^4}{4}$

Isosceles Triangle, depth h, base b: $I = \frac{bh^3}{36}$ (note centroid is at h/3 above the base)

Parallel axis theorem:

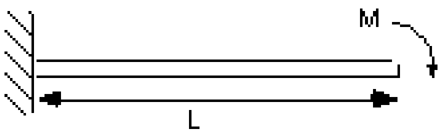
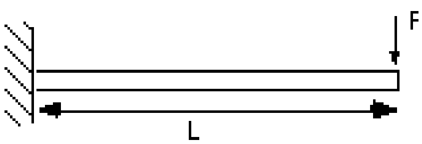
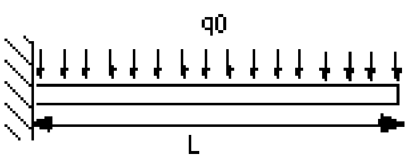
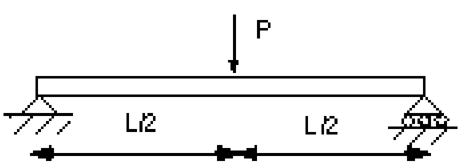
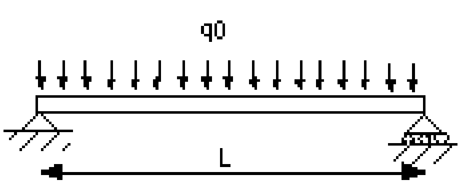
If the second moment of area of a section, area A, about an axis is I then the second moment of area I′ about a parallel axis, a perpendicular distance d away from the original axis is given by:

$$I' = I + Ad^2$$

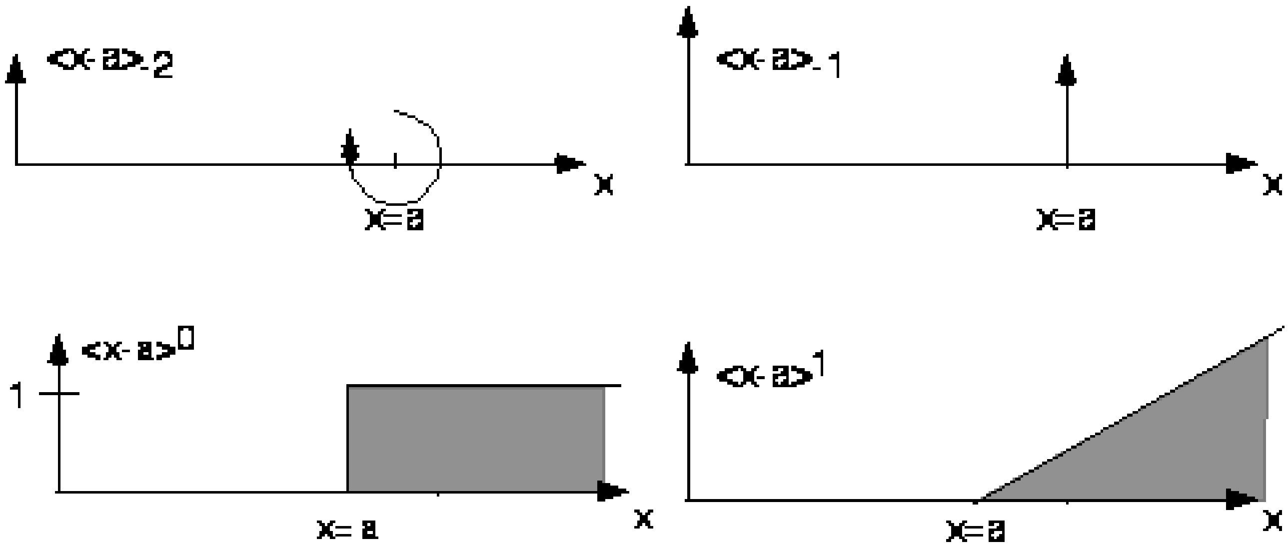
First moment of area
The first moment of area of a section between a height z from the neutral plane and the top surface (outer ligament) of the section is given by:

$$Q = \int_{A,z}^{h/2} z\,dA$$

Standard solutions for deflections of beams under commonly encountered loading

Configuration	End slope dw/dx (x=L)	End deflection, w(L)	Central deflection, w(L/2)
	$\frac{ML}{EI}$	$\frac{ML^2}{2EI}$	
	$\frac{PL^2}{2EI}$	$\frac{PL^3}{3EI}$	
	$\frac{q_0L^3}{6EI}$	$\frac{q_0L^4}{8EI}$	
	$\frac{PL^2}{16EI}$		$\frac{PL^3}{48EI}$
	$\frac{q_0L^3}{24EI}$		$\frac{5q_0L^4}{384EI}$

Singularity functions



Integration of singularity functions: $\int_{-\infty}^{x} \langle x-a \rangle^{n} dx = \dfrac{\langle x-a \rangle^{n+1}}{n+1}, \quad n \geq 0$

$$\int_{-\infty}^{x} \langle x-a \rangle_{-2} \, dx = \langle x-a \rangle_{-1} \qquad \int_{-\infty}^{x} \langle x-a \rangle_{-1} \, dx = \langle x-a \rangle^{0}$$

in a plane perpendicular to a principal direction.