Fluids – Lecture 21 Notes

1. Airfoil Polar Relations

Airfoil Polar Relations

Wing Loading

For an aircraft in steady level flight, we have that lift equals weight.

$$L = \frac{1}{2}\rho V^2 S\, C_L = mg$$

$$\frac{1}{2}\rho V^2 = \frac{mg}{S}\frac{1}{C_L}$$

The ratio mg/S is called the *wing loading* and has the units of force/area, or pressure. The lift coefficient C_L can be interpreted here as the constant of proportionality between the wing loading and the dynamic pressure. The airspeed is given explicitly by

$$V = \sqrt{\frac{mg}{S}\frac{2}{\rho\, C_L}}$$

An important performance measure of many airplanes is their speed range, or their max/min speed ratio. From the above equations we see that V is maximum when C_L is minimum, and vice versa.

$$\frac{1}{2}\rho V_{\rm min}^2 = \frac{mg}{S}\frac{1}{C_{L_{\rm max}}}$$

We can therefore form the ratio

$$\frac{V_{\rm max}}{V_{\rm min}} = \sqrt{\frac{C_{L_{\rm min}}}{C_{L_{\rm max}}}}$$

so a large speed ratio requires a very large C_L ratio. Efficient flight at any particular C_L, whether large or small, requires that the corresponding C_D is acceptably low. The acceptable C_L range can be discerned on a $C_D(C_L)$ drag polar of the aircraft. A major component of this is the wing airfoil's $c_d(c_\ell)$ drag polar.

Sample airfoil polars

The figures show two airfoils for RC aircraft.

Dragonfly. This airfoil is used on the Dragonfly light electric sport aircraft.

AG44ct. This airfoil is used on high-performance composite RC sailplanes.

The most striking difference in the two airfoils is the camber:

Dragonfly:	7.3% camber
AG44ct(-2)	1.9% camber
AG44ct(+4)	3.4% camber

The main consequence is for the minimum flyable C_L. The Dragonfly airfoil cannot operate much below $C_{L_{\rm min}} = 0.5$ without incurring a massive drag increase (due to the bottom surface stalling). The AG44ct(-2) in contrast can operate very near zero C_L, with $C_{L_{\rm min}} = 0.05$ being a practical lower limit. The maximum usable lift coefficients are roughly $C_{L_{\max}} = 1.3$ for the Dragonfly, and $C_{L_{\max}} = 0.85$ for the RC sailplane with maximum camber flap deployed.

Both the Dragonfly and a typical RC sailplane have comparable wing loadings:

$$\frac{mg}{S} \simeq 15\,\mathrm{Pa}$$

The corresponding minimum and maximum speeds for the Dragonfly are

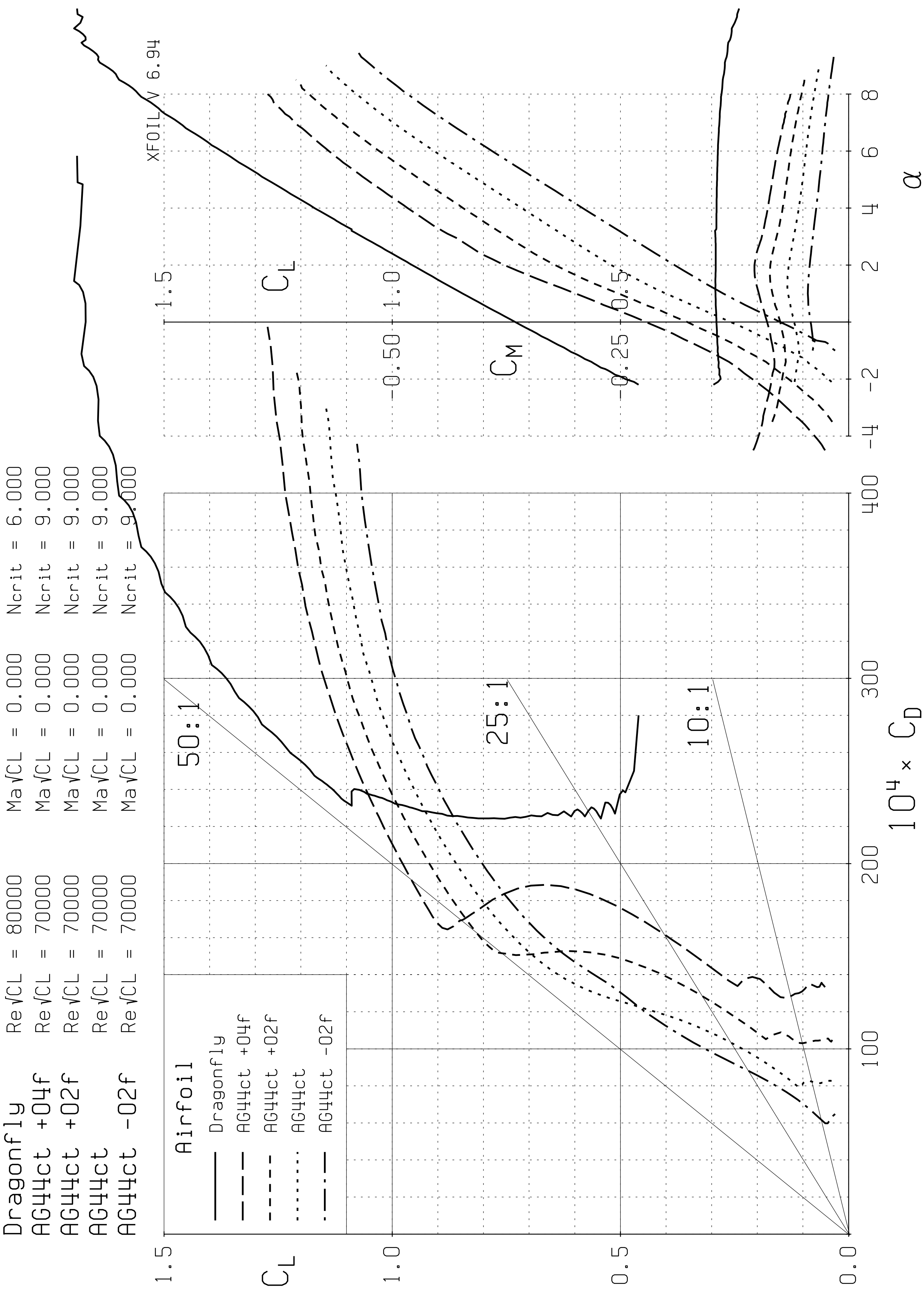
$$\begin{aligned} V_{\min} &= 4.3\,\mathrm{m/s} = 9.7\,\mathrm{mph} \qquad &\text{(Dragonfly)} \\ V_{\max} &= 7.0\,\mathrm{m/s} = 15.6\,\mathrm{mph} \qquad &\text{(Dragonfly)} \end{aligned}$$

For the RC saiplane they are

$$\begin{aligned} V_{\min} &= 5.3\,\mathrm{m/s} = 12.0\,\mathrm{mph} \qquad &\text{(RC sailplane)} \\ V_{\max} &= 22\,\mathrm{m/s} = 49.3\,\mathrm{mph} \qquad &\text{(RC sailplane)} \end{aligned}$$

Note the much larger speed range of the RC sailplane, which is important for fast ranging in search of thermals, possibly against the wind. The sport Dragonfly has no such performance requirement, and its narrow speed range is not a serious handicap.


-2 flap
+4 flap


Dragonfly Re√CL = 80000 Ma√CL = 0.000 Ncrit = 6.000
AG44ct +04f Re√CL = 70000 Ma√CL = 0.000 Ncrit = 9.000
AG44ct +02f Re√CL = 70000 Ma√CL = 0.000 Ncrit = 9.000
AG44ct Re√CL = 70000 Ma√CL = 0.000 Ncrit = 9.000
AG44ct -02f Re√CL = 70000 Ma√CL = 0.000 Ncrit = 9.000
Airfoil
Dragonfly
AG44ct +04f
AG44ct +02f
AG44ct
AG44ct -02f
50:1
25:1
10:1
C_L
C_M
10^4 × C_D
α
XFOIL V 6.94
0.0
0.5
1.0
1.5
-0.25
-0.50
100
200
300
400
-4
-2
2
4
6
8