Fluids – Lecture 20 Notes

   1. Airfoils – Detailed Look

Reading: `http://www.av8n.com/how/htm/`   Sections 3.1–3.3   (optional)

Airfoils – Detailed Look

Flow curvature and pressure gradients

The pressures acting on an airfoil are determined by the airfoil's overall shape and the angle of attack. However, it's useful to examine how local pressures are approximately affected by local geometry, and the surface curvature in particular.

Consider a location near the airfoil surface, ignoring the thin boundary layer. The local flow speed is V, the local streamline radius of curvature is R. Another equivalent way to define the curvature is $\kappa = 1/R = d\theta/ds$, where θ is the inclination angle of the surface or streamline, and s is the arc length. Positive κ is defined to be concave up as shown.
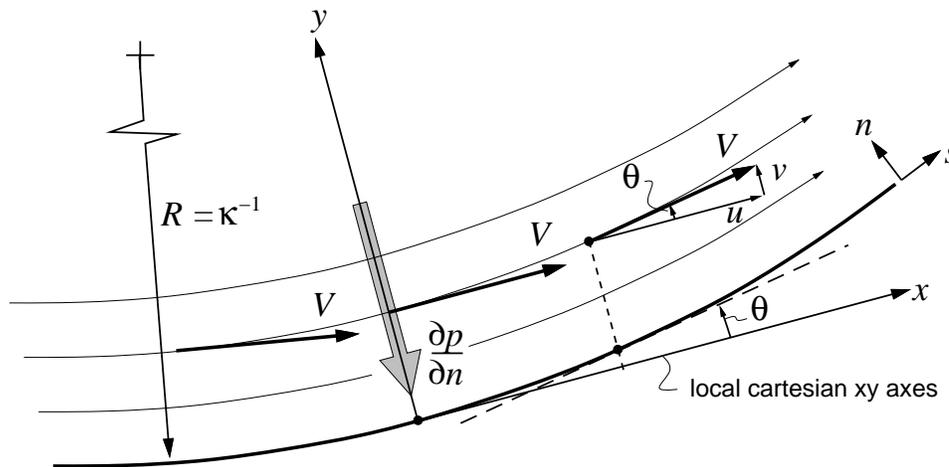
To determine how the pressure varies normal to the surface, we align local xy axes tangent and normal to the surface, and employ the y-momentum equation, with the viscous forces neglected.

$$\frac{\partial p}{\partial y} \;=\; -\rho u\,\frac{\partial v}{\partial x} \;-\; \rho v\,\frac{\partial v}{\partial y}$$

The Cartesian velocity components are related to the speed and the surface angle as follows.

$$u \;=\; V\,\cos\theta \qquad , \qquad\qquad v \;=\; V\,\sin\theta$$

At the origin where $\theta = 0$, we then have

$$v = 0 \qquad , \qquad\quad u = V \qquad , \qquad\quad \frac{\partial v}{\partial x} \;=\; \frac{\partial V}{\partial x}\,\sin\theta \;+\; V\,\cos\theta\,\frac{\partial \theta}{\partial x} \;=\; V\,\kappa$$

Therefore, the normal pressure gradient along $y = n$ is

$$\frac{\partial p}{\partial n} \;=\; -\rho\, V^2 \kappa$$

This is the *normal-momentum equation*, sometimes also called the *centrifugal formula.* It describes the physical requirement that there must be a transverse pressure gradient to force fluid to flow along a curved streamline. It is valid for inviscid flows, at any Mach number.

Implications for surface pressures

Because of the influence on normal pressure gradients, changes in surface curvature are expected to cause changes in surface pressure. If a common reference pressure exists away from the wall, a concave corvature will produce a higher pressure towards the wall, while a convex curvature will produce a lower pressure towards the wall. The figure below illustrates the situation for a simple bump. The "+" and "-" symbols indicate expected changes in pressure.
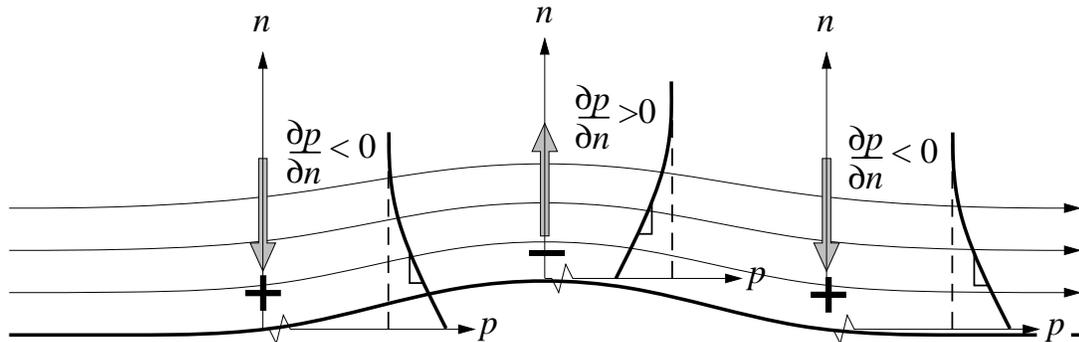

The curvature/pressure-gradient relation also indicates the pressures which can be expected on the surface of a body such as an airfoil. Examination of the streamline curvatures indicates that for a symmetric airfoil at zero angle of attack, higher pressure is expected at the leading and trailing edges, while lower pressure is expected along the sides.
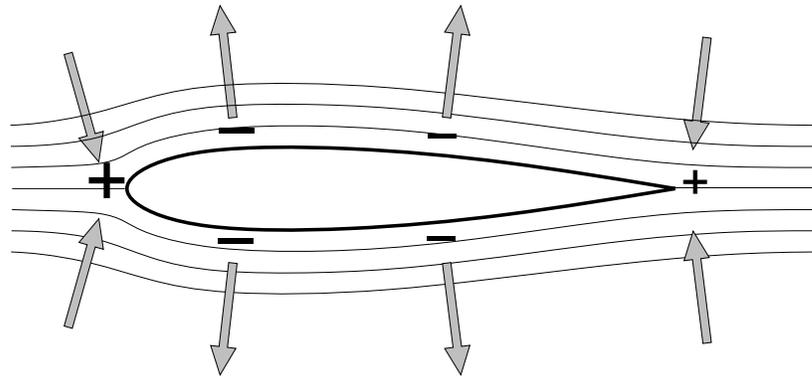
For the same symmetrical airfoil at an angle of attack, the streamline pattern and the pressures near the leading edge are now considerably different. The stagnation point moves under the leading edge, and a strongly reduced pressure, called a "leading edge spike", forms at the leading edge point itself.
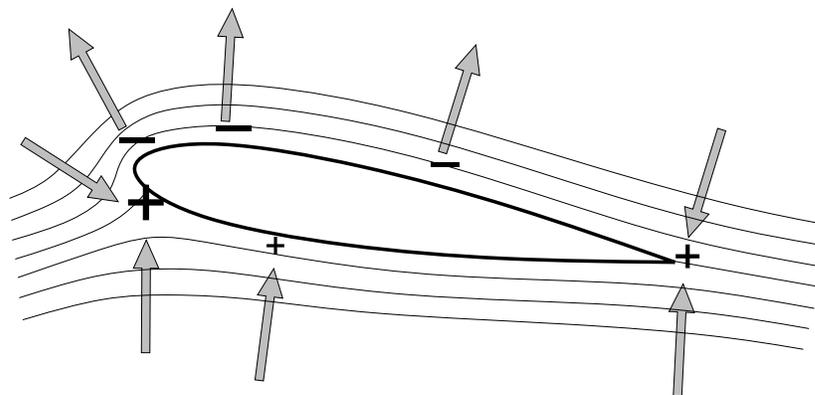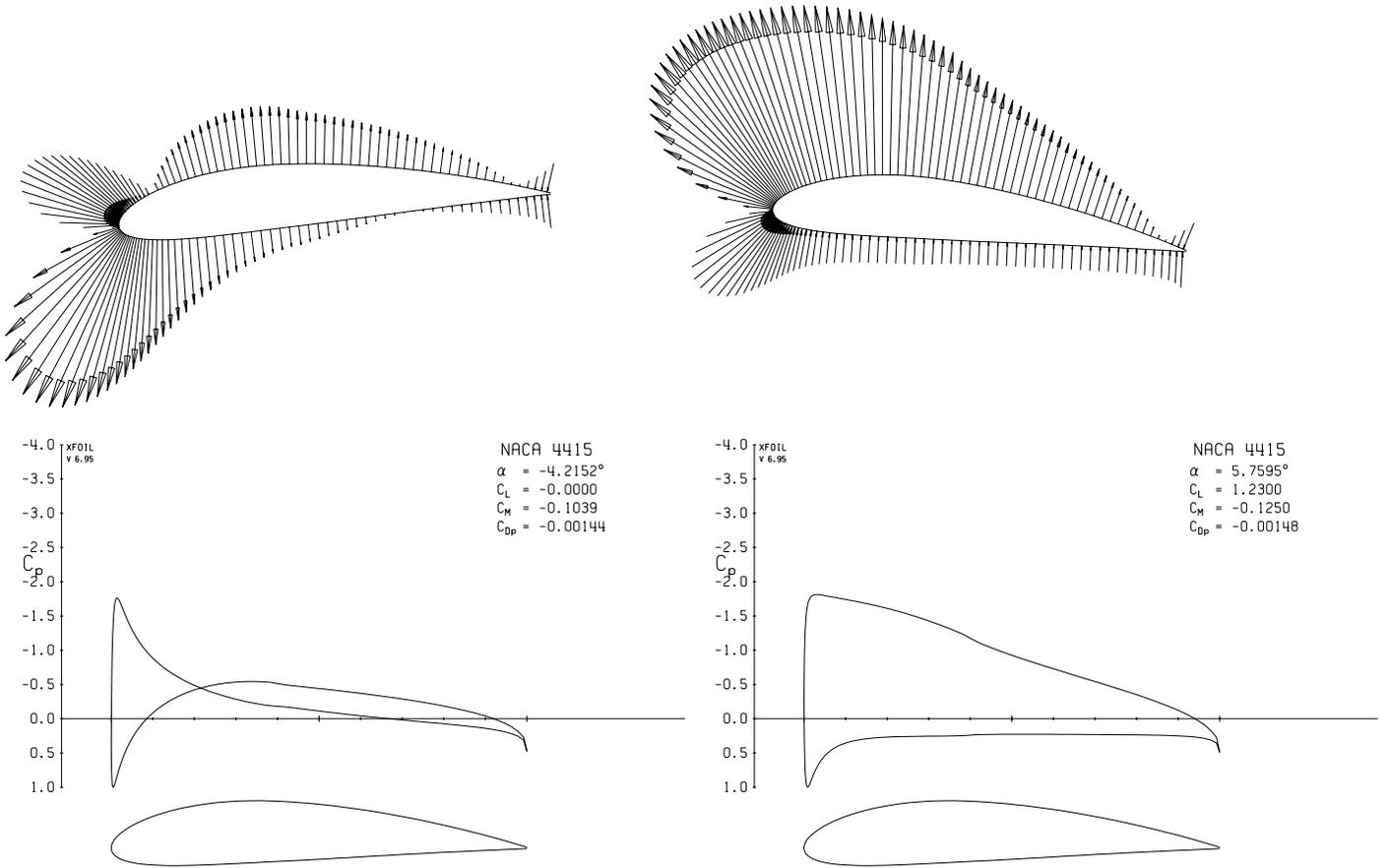
The actual surface pressure force vectors $-C_p \hat{n}$ are shown for the NACA 0015 airfoil, at $\alpha = 0°$ ($c_\ell = 0$), and $\alpha = 10°$ ($c_\ell = 1.23$). The $C_p(x)$ distributions are also shown plotted.
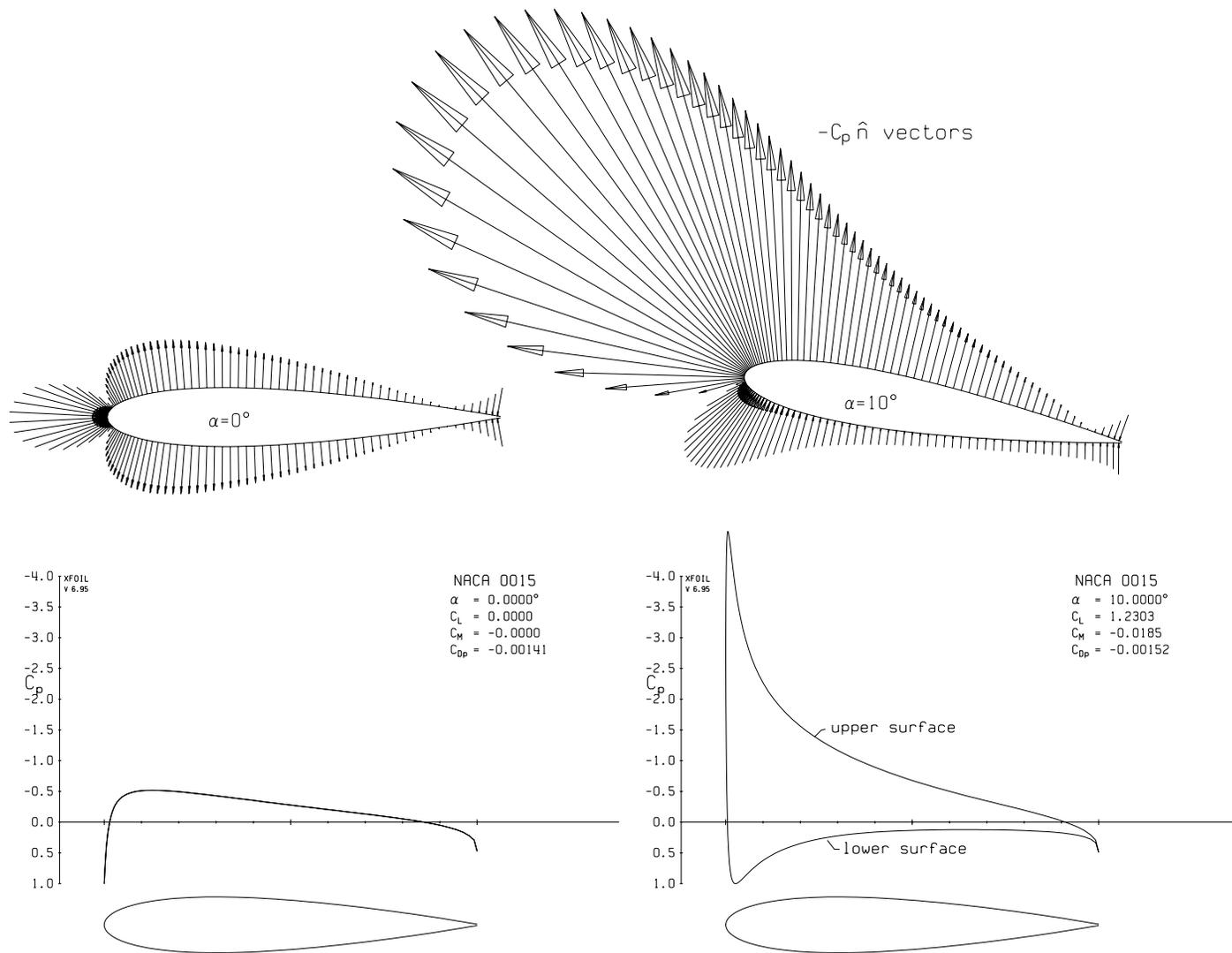
These results were computed using a panel method, and therefore correspond to inviscid irrotational incompressible flow. The drag is predicted to be zero (d'Alembert's Paradox), and the possibility of boundary layer separation is ignored. Despite these limitations, the calculations are useful in that they simply reveal the intense pressure spike, which is known to promote separation of the upper surface boundary layer, and thus degrades the airfoil's stall resistance. A corrective redesign of the airfoil would normally be undertaken if the leading edge spike is deemed to be too strong.

Use of camber
An effective way to reduce the intensity of the leading edge spike is to add *camber* to the airfoil. The NACA 4415 airfoil has the same 15% maximum thickness (relative to chord) as the 0015, but it has a nonzero 4% maximum camber. The figures below show the cambered NACA 4415 airfoil at the same same $c_\ell = 0$ and $c_\ell = 1.23$ as in the NACA 0015 case (comparing at the same lift or c_ℓ is more meaningful than comparing at the same α).

NACA 4415
$\alpha = -4.2152°$
$C_L = -0.0000$
$C_M = -0.1039$
$C_{Dp} = -0.00144$

C_p

-4.0
-3.5
-3.0
-2.5
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0

NACA 4415
$\alpha = 5.7595°$
$C_L = 1.2300$
$C_M = -0.1250$
$C_{Dp} = -0.00148$

C_p

-4.0
-3.5
-3.0
-2.5
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0

The leading edge spike at the high angle of attack is indeed reduced considerably. The low angle of attack case now has a "negative" spike on the bottom surface, but this is much weaker and appears tolerable.

Although camber is seen to be attractive in the case above, too much camber is usually detrimental. The figure below shows the NACA 8415 at the same $c_\ell = 0$ and $c_\ell = 1.23$ conditions. The intense spike on the bottom surface shows the drawback of using the excessive 8% camber – the low c_ℓ (high speed) condition is likely to have excessive drag. Selection of the ideal amount of camber is a major design choice for the airplane designer.