

## Lecture F18 Mud: Vortex, Lift

(38 respondents)

1. **What's the difference between  $r$  and  $R$ ?** (1 student)

$r$  is a general radius, as in  $r$ - $\theta$  polar coordinates.  $R$  is the specific cylinder's radius.

2. **How can we define  $C$  as  $-\Gamma/2\pi$ ?** (1 student)

For a vortex defined by  $V_\theta = C/r$ , we calculate its circulation to be  $\Gamma = -2\pi C$ . So  $C$  and  $\Gamma$  are effectively equivalent, since they differ only by the constant factor of  $-2\pi$ . We can use them interchangeably. Since  $\Gamma$  is more aerodynamically significant (directly related to vorticity, lift, etc.), we choose to use  $\Gamma$  instead of  $C$  to define the vortex:  $V_\theta = -\Gamma/(2\pi r)$ .

3. **I don't understand  $\Gamma$ .** (1 student)

I don't know how to explain it better without just repeating the lecture material. Maybe try reading Anderson 2.13 again? One suggestion is to clearly distinguish the definition from the implications. Two equivalent definitions are

$$\Gamma \equiv - \oint \vec{V} \cdot d\vec{s} \quad \text{or} \quad \Gamma \equiv - \iint \vec{\xi} \cdot \hat{n} dA$$

Some implications are:

$\Gamma = 0$  for any circuit in irrotational flow not enclosing a body.

$\Gamma = L'/\rho V_\infty$  for any circuit enclosing an airfoil with lift  $L'$ .

4. **For the vortex flowfield, is  $\Gamma$  nonzero only if origin is enclosed in the  $xy$  plane, as opposed to the  $yz$  plane?** (1 student)

First of all, the vortex flow is a 2-D flow in the  $xy$  or  $r\theta$  plane. So in the real 3-D world it's actually a "roller" threaded on the  $z$  axis. We can take any circuit which encloses the  $z$  axis and get the same  $\Gamma$  value. The circuit does not have to lie flat in the  $xy$  plane.

5. **In the notes, the total stream function for the uniform flow + doublet + vortex has  $-(\Gamma/2\pi) \ln r$  for the vortex part. Shouldn't that be  $+(\Gamma/2\pi) \ln r$ ?** (1 student)

You are correct. It's a typo which has been corrected in the web notes.

6. **How can  $D' = 0$ ?** (1 student)

It can be seen to be zero by symmetry. The speed  $|V_\theta(\theta)|$  and hence the pressure  $p(\theta)$  are always the same on corresponding left and right points, for any combination of  $V_\infty$ , doublet strength  $\kappa$ , and vortex strength  $\Gamma$ . So all the  $x$ -components forces which produce  $D'$  add up to exactly zero.

7. **When did we neglect viscosity?** (1 student)

We neglected viscosity way back when we assumed irrotational flow, and started using  $\vec{V} = \nabla\phi$  and the Bernoulli equation. Once you go this route you are destined to have  $D' = 0$  for any closed 2-D body (d'Alembert's Paradox).

We acknowledge that the inviscid assumption is quite bad for a bluff body with flow separation. But it is very good for streamlined bodies like airfoils, at least if mainly  $L'$  and  $M'$  are of interest.

8. **On the board you wrote**

$$C_p = 1 - \frac{V^2}{V_\infty^2} = 1 - \frac{V_r^2 + V_\theta^2}{V_\infty^2}$$

**Shouldn't that be  $(V_r + V_\theta)^2$ ?** (1 student)

No. I had it correct.  $V$  is the magnitude of the total velocity vector, and so  $V^2 = V_r^2 + V_\theta^2$  by Pythagoras.

9. **Why is  $\oint p \hat{n} dA = \oint (p - p_\infty) \hat{n} dA$ ?** (1 student)

We can add any constant ( $p_\infty$  in this case) to this integrand and not change the result. This is because of the following math identity for any closed integration contour:

$$\oint (\text{constant}) \hat{n} dA = 0$$

The physical interpretation is that a constant pressure ( $p_\infty$  in this case) cannot exert a net force on a closed body.

10. **What's the difference between lifting and non-lifting flows in the way you model them?** (1 student)

Good question! A non-lifting flow can be modeled by sources alone, although vortices can also be used. A lifting-flow model must include vortices in one way or another, since to get lift you must have  $\Gamma \neq 0$ , and have  $\Gamma \neq 0$  you must have vortices present. Sources always have a zero circulation and hence by themselves cannot represent a lifting flow.

11. **Lost in the math. Do we need to understand all that algebra?** (4 students)

I admit the math is relatively complicated. But there's no way to effectively describe fluid flows without it. I try to use physical pictures when possible, but these are a qualitative aid, not a substitute for the math.

Understanding the starting assumptions and interpreting the final result are vastly more important than being able to recite the algebra in between.

12. **No mud** (13 students)