Lecture F14 Mud: Helmholtz, Laplace's Equation
(33 respondents)

1. **What is the physical meaning of $\nabla^2 \phi$, or ∇^2 in general?** (2 students)
   In the case of fluid flow, $\nabla^2 \phi$ is the same as $\nabla \cdot \vec{V}$, or the velocity divergence. Also, for 2-D flow, $\nabla^2 \psi$ is the same as $\nabla \times \vec{V}$, or the vorticity. Laplace's equation also appears in a great number of other situations in math and physics, such as in Electricity and Magnetism.

2. **Can we use Laplace Transforms to solve for streamline shapes?** (1 student)
   No. Laplace's equation and Laplace transforms are totally unrelated. Laplace himself dabbled in lots of different stuff.

3. **How do you actually solve $\nabla^2 \phi = 0$?** (1 student)
   There are lots of different techniques, many employing superposition. In aerodynamics, one class of techniques are called *panel methods* which we will discuss later.

4. **What's the Helmoltz equation used for?** (1 student)
   A major use is to prove that aerodynamic flows are irrotational, like we did in class.

5. **Can we plot streamlines by plotting $\psi(x, y) = $ constant?** (1 student)
   Yep.

6. **What's the difference between ϕ and ψ?** (2 students)
   Each is used to generate a velocity field, but in two different ways. I suggest reviewing the past few lectures.

7. **How do you define where the boundary layer starts?** (1 student)
   One way to define the edge of a boundary layer is: "Normal distance from the wall beyond which the vorticity is below some very small threshold." At the edge the vorticity decreases extremely fast, roughly as e^{-n^2}, so the precise threshold level is not too important.

8. **How can the wall be another streamline when it has boundary layers on it?** (1 student)
   Good question. When solving $\nabla^2 \phi = 0$ or $\nabla^2 \psi = 0$ for the flow around a body we assume that the boundary layers are negligibly thin. This is by necessity, since we've assumed that viscous effects are neglible. In flow situations where the viscous effects are actually <u>not</u> neglible, such as in the flow around a cylinder in the Fluids Lab, the idealized inviscid flow that we predict from the solution of $\nabla^2 \phi = 0$ will not be very realistic. Using your data you will compare the ideal inviscid flow to the actual flow.

9. **Why does $\partial\phi/\partial n = 0$ mean that the flow is perpendicular to $\hat{n}$?** (1 student)
   $\partial\phi/\partial n = 0$ is the same as $\vec{V} \cdot \hat{n} = 0$, which means the normal velocity component V_n is zero. This is the definition of "perpendicularity".

10. **What's the difference in boundary conditions between infinity and at a wall?** (1 student)
    They specify different things. $\partial\phi/\partial x = V_\infty$ states that the horizontal velocity is equal to V_∞. $\partial\phi/\partial n = 0$ states than the normal velocity (V_n in an earlier lecture) is zero.

11. **The superposition example was like the flow around a torpedo nose. How is that different than the flow around a missile nose?** (1 student)
I picked a torpedo since the nose shape in the example is sort of rounded. To get the flow to take on a pointy nose shape like on a missle requires superimposing something else besides a single source.

12. **No mud** (19 students)