Lecture F12 Mud: Stream Function, Potential Function
(25 respondents)

1. **The book claims "$\bar{\psi}_2 = \bar{\psi}_1$ represents mass flow perpendicular to the page".**
   Isn't 2-D flow in the plane of the page? (1 student)
   It actually says "mass flow per unit depth perpendicular to the page". The "perpendicular" refers to the unit depth, not to the flow.

2. **What's the difference between $\bar{\psi}$ and ψ?** (2 students)
   $\bar{\psi}$ is the most general type of stream function, and gives the mass flux components:

$$\partial\bar{\psi}/\partial y = \rho u \qquad\qquad -\partial\bar{\psi}/\partial x = \rho v$$

   For low speed flows where ρ <u>is</u> a constant, it is convenient to absorb the constant ρ factor into the stream function by defining $\psi = \bar{\psi}/\rho$. We now have:

$$\partial\psi/\partial y = u \qquad\qquad -\partial\psi/\partial x = v$$

   It's important to remember that ψ can be used only for low speed flows, while $\bar{\psi}$ has no such restriction.

   The difference $\bar{\psi}_2 - \bar{\psi}_1$ gives the *mass flow* between two streamlines, while $\psi_2 - \psi_1$ gives the *volume flow* between two streamlines (but only in low-speed flow).

3. **Where does $\psi(x,y) = \ln\sqrt{x^2 + y^2}$ come from?** (1 student)
   I just made it up. The neat thing about a stream function is that no matter how complicated $\psi(x,y)$ might get, the resulting $u(x,y)$ and $v(x,y)$ that you get from it satisfy the mass continuity equation

$$\nabla \cdot \vec{V} \equiv \partial u/\partial x + \partial v/\partial y = 0$$

   and hence represent a physically possible flow.

4. **Is there are graphical relation between streamlines and stream function lines?** (1 student)
   Lines of constant stream function are the same as streamlines.

5. **I don't understand how the funnel-shaped ψ and the spiral-shaped ϕ represent the same vortex flow?** (1 student)
   Both functions produce the same $u(x,y)$ and $v(x,y)$, and hence they both represent the same physical flowfield.

6. **Will be dealing mostly with 2-D flows?** (1 student)
   The approach in Unified Fluids is to present concepts in the simplest way possible, so that the understanding isn't lost in unnecessary complexity. So we use the smallest number of spatial dimensions in the examples and applications. Sometimes 1-D, usually 2-D, and occasionally 3-D if it's unavoidable.

7. **What's the difference between ψ and ϕ?** (1 student)
They are alternative ways to define the velocity field $u(x, y)$ and $v(x, y)$. There are important differences also. For example, ψ is usable only in 2-D, while ϕ easily extends to 3-D with minimal complication.

8. **Is there a physical interpretation of $\bar{\psi}$, ψ, ϕ?** (1 student)
$\bar{psi}$ has several interpretations: streamlines, mass flow, as presented in the notes. ϕ is a bit harder to interpret.

9. **Why would we use ψ or ϕ?** (1 student)
When applicable, they almost always produce a tremendous mathematical simplification of a fluid flow problem. This makes solving the equations much easier and/or faster.

10. **Are there conditions under which these methods work?** (1 student)
Yes. The main restrictions are: ψ and $\bar{\psi}$ are usable only in 2-D
ϕ is usable only for irrotational flows.

11. **Seems like we're learning only endless equations and symbols. Where's the justification?** (1 student)
So far we've focused almost entirely on "tools and concepts". Applications will come next. Anderson lays out this strategy.

12. **Can you give a numerical example of calculating a directional derivative?** (1 student)
A directional derivative is normally computed using the dot product.

$$\partial\phi/\partial n \;=\; \nabla\phi \cdot \hat{n}$$

For example, say
$$\phi(x, y) = -\arctan(y/x)$$
and we want to know $\partial\phi/\partial n$ at the point $(x, y) = (0, 1)$, along a line tilted 45° up from horizontal. Along this direction

$$\hat{n} \;=\; \frac{1}{\sqrt{2}}\hat{\imath} \;+\; \frac{1}{\sqrt{2}}\hat{\jmath}$$

and the gradient at the chosen point is

$$\nabla\phi \;=\; 1\,\hat{\imath} \;+\; 0\,\hat{\jmath}$$

Hence,
$$\partial\phi/\partial n \;=\; \nabla\phi \cdot \hat{n} \;=\; \frac{1}{\sqrt{2}}$$

13. **No mud** (10 students)