Fluids – Lecture 8 Notes

1. Streamlines
2. Pathlines
3. Streaklines

Reading: Anderson 2.11

Three types of fluid element trajectories are defined: Streamlines, Pathlines, and Streaklines. They are all equivalent for steady flows, but differ conceptually for unsteady flows.

Streamlines

Streamline equations

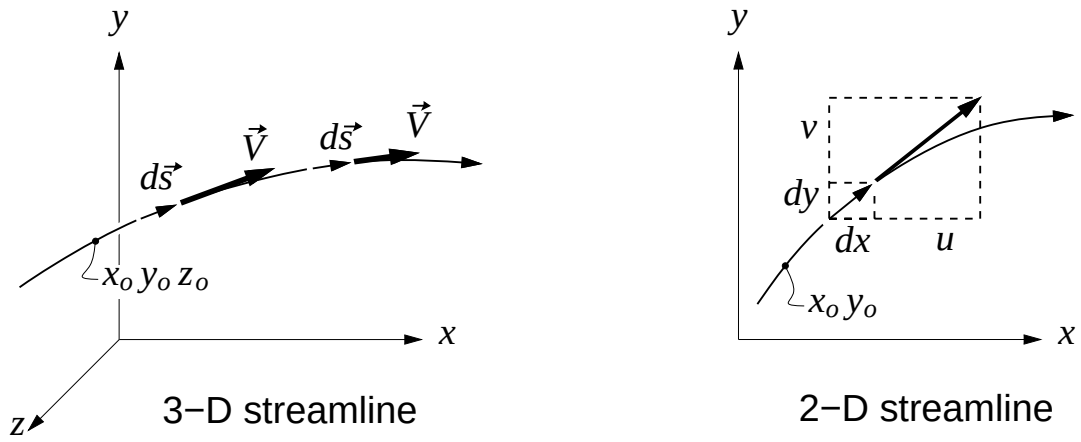
A *streamline* is defined as a line which is everywhere parallel to the local velocity vector $\vec{V}(x,y,z,t) = u\,\hat{\imath} + v\,\hat{\jmath} + w\,\hat{k}$. Define

$$d\vec{s} \;=\; dx\,\hat{\imath} \;+\; dy\,\hat{\jmath} \;+\; dz\,\hat{k}$$

as an infinitesimal arc-length vector along the streamline. Since this is parallel to $\vec{V}$, we must have

$$\begin{aligned} d\vec{s} \times \vec{V} &= 0 \\ (w\,dy - v\,dz)\,\hat{\imath} \;+\; (u\,dz - w\,dx)\,\hat{\jmath} \;+\; (v\,dx - u\,dy)\,\hat{k} &= 0 \end{aligned}$$

Separately setting each component to zero gives three differential equations which define the streamline. The three velocity components u, v, w, must be given as functions of x, y, z before these equations can be integrated. To set the constants of integration, it is sufficient to specify some point x_o, y_o, z_o through which the streamline passes,



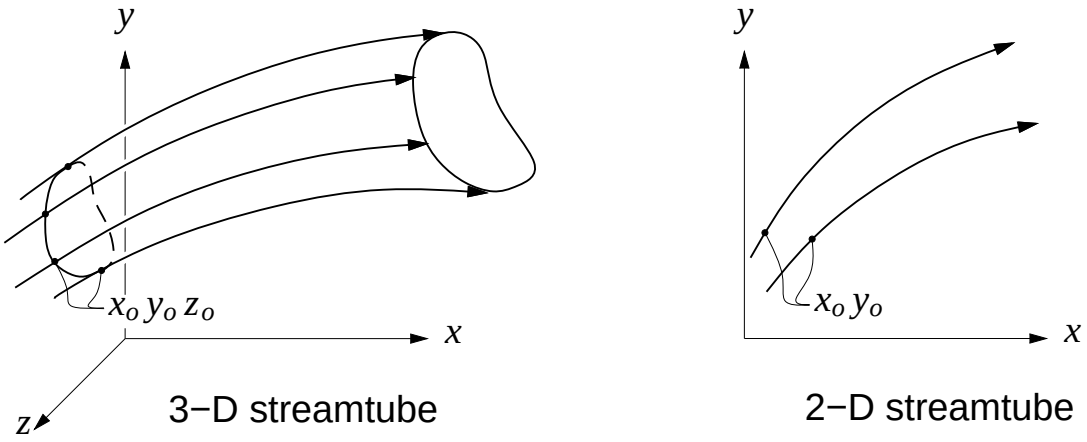
In 2-D we have $dz = 0$ and $w = 0$, and only the $\hat{k}$ component of the equation above is non-trivial. It can be written as an Ordinary Differential Equation for the streamline shape $y(x)$.

$$\frac{dy}{dx} \;=\; \frac{v}{u}$$

Again, $u(x,y)$ and $v(x,y)$ must be given to allow integration, and x_o, y_o must be given to set the integration constants. In a numerical integration, x_o, y_o would serve as the initial values.

Streamtubes

Consider a set of x_o, y_o, z_o points arranged in a closed loop. The streamlines passing through all these points form the surface of a *streamtube*. Because there is no flow across the surface, each cross-section of the streamtube carries the same mass flow. So the streamtube is equivalent to a channel flow embedded in the rest of the flowfield.



In 2-D, a streamtube is defined by two streamlines passing through two specified x_o, y_o points. The flow between these two streamlines carries the same mass flow/span at each cross-section, and can be considered as a 2-D channel flow embedded in the rest of the flowfield.

Pathlines

The *pathline* of a fluid element A is simply the path it takes through space as a function of time. An example of a pathline is the trajectory taken by one puff of smoke which is carried by the steady or unsteady wind. This path is fully described by the three position functions $x_A(t)$, $y_A(t)$, $z_A(t)$, which can be computed by integrating the three velocity-field components $u(x, y, z, t)$, $v(x, y, z, t)$, $w(x, y, z, t)$ along the path. The integration is started at time t_o, from the element's initial position x_o, y_o, z_o (e.g. the smoke release point), and proceeds to some later time t.

$$
\begin{aligned}
x_A(t) &= x_o + \int_{t_o}^{t} u\left(x_A(\tau), y_A(\tau), z_A(\tau), \tau\right)\, d\tau \\
y_A(t) &= y_o + \int_{t_o}^{t} v\left(x_A(\tau), y_A(\tau), z_A(\tau), \tau\right)\, d\tau \\
z_A(t) &= z_o + \int_{t_o}^{t} w\left(x_A(\tau), y_A(\tau), z_A(\tau), \tau\right)\, d\tau
\end{aligned}
$$

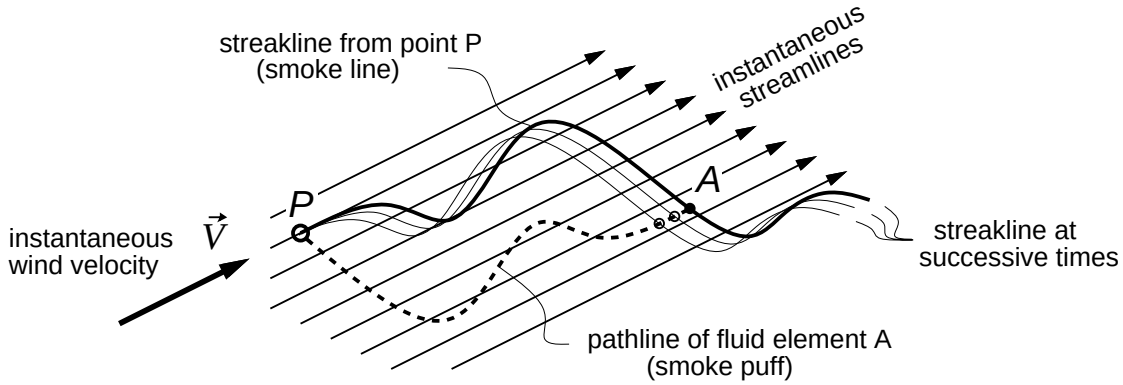
The dummy variable of integration τ runs from t_o to t.

Streaklines

A *streakline* is associated with a particular point P in space which has the fluid moving past it. All points which pass through this point are said to form the streakline of point P. An example of a streakline is the continuous line of smoke emitted by a chimney at point P, which will have some curved shape if the wind has a time-varying direction.

Unlike a pathline, which involves the motion of only one fluid element A in time, a streakline involves the motion of all the fluid elements along its length. Hence, the trajectory equations for a pathline are applied to all the fluid elements defining the streakline.

The figure below illustrates streamlines, pathlines, and streaklines for the case of a smoke being continuously emitted by a chimney at point P, in the presence of a shifting wind. One particular smoke puff A is also identified. The figure corresponds to a snapshot when the wind everywhere is along one particular direction.



In a steady flow, streamlines, pathlines, and streaklines all cooincide. In this example they would all be marked by the smoke line.