NAME Solutions

# Unified Quiz 3S

November 7, 2003

One 81/2” x 11” sheet (two sides) of notes allowed.

Calculators allowed.

Calculators may NOT be used for eigenvalue / eigenvector calculation or for solving matrix equations

No books allowed.

- Put your name on each page of the exam.
- Read all questions carefully.
- Do all work for each problem on the two pages provided.
- Show intermediate results.
- Explain your work --- don’t just write equations. Any problem (except multiple choice) without an explanation can receive no better than a “B” grade.
- Partial credit will be given, but only when the intermediate results and explanations are clear.
- Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.
- Show appropriate units with your final answers.
- Box your final answers.

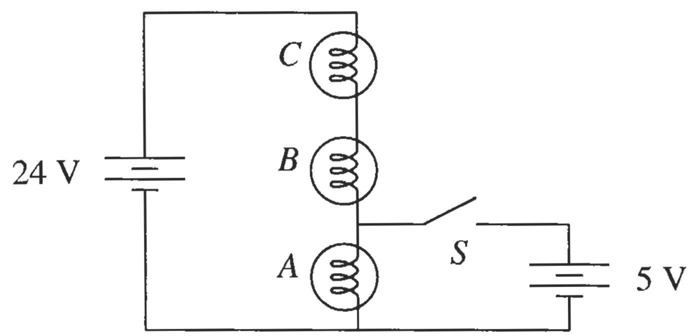
## Exam Scoring

| | |
|---|---|
| #1 (25%) | |
| #2 (20%) | |
| #3 (30%) | |
| #4 (25%) | |
| Total | Joe B = 75 |

Unified Quiz 3S
November 7, 2003

NAME SOLUTIONS

## PROBLEM #1 (25%)




A circuit consists of three identical light bulbs, two batteries, and a switch, as shown above. When the switch *S* is closed, do the following increase, decrease, or stay the same? Circle one for each part.

1. The intensities of bulbs *B* and *C*

   increase     decrease     stay the same

2. The intensity of bulb *A*

   increases     decreases     stays the same

3. The current flow in the 24 V battery

   increases     decreases     stays the same

4. The power supplied by the 24 V battery to the circuit

   increases     decreases     stays the same

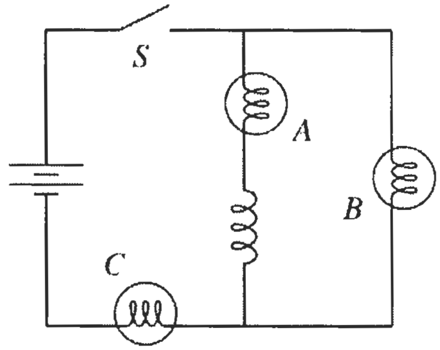
5. The power supplied by the 5 V battery to the circuit

   increases     decreases     stays the same

Unified Quiz 3S
November 7, 2003

NAME SOLUTIONS

**PROBLEM #1 (continued)**




Consider the circuit above with three identical light bulbs, a battery, an inductor, and a switch. Initially, the switch is open, and has been open for a long time. The switch is then closed.

6. Immediately after the switch is closed, the intensity of bulb *A* is

   less than    greater than    the same as

   the intensity of bulb *B*.

7. Immediately after the switch is closed, the intensity of bulb *C* is

   less than    greater than    the same as

   the intensity of bulb *B*.

8. After the switch has closed for a long time, the intensity of bulb *B* is

   less than    greater than    the same as

   the intensity of bulb *A*.

9. After the switch has closed for a long time, the intensity of bulb *C* is

   less than    greater than    the same as

   the intensity of bulb *A*.

10. After the switch has been closed for a long time, the switch is then opened. Immediately after the switch is opened, the intensity of bulb *B* is

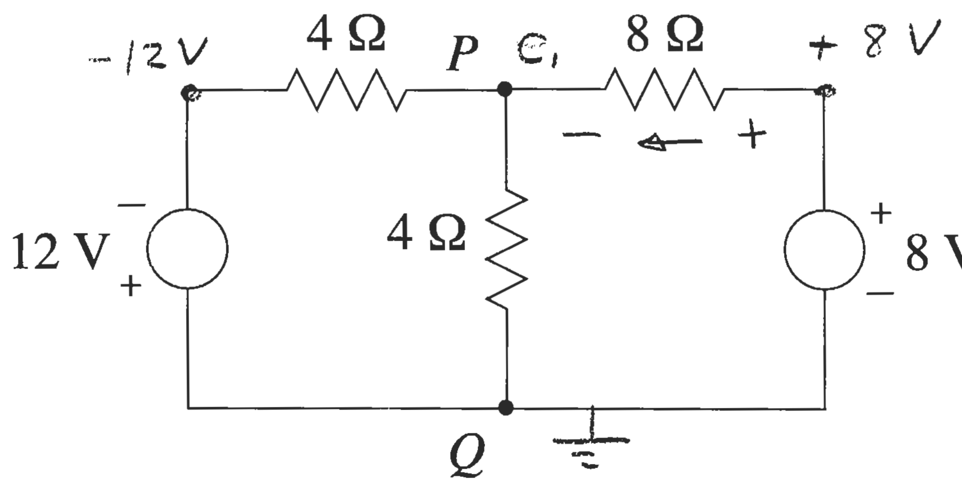
    less than    greater than    the same as

    the intensity of bulb *A*.

Unified Quiz 3S
November 7, 2003

NAME Solutions

**PROBLEM #2 (20%)**




For the circuit above, calculate:

(a) The current in the 8 Ω resistor. Make sure that you specify the direction of the current.

(b) The potential difference between points *P* and *Q*. Make sure you specify which node is at the higher potential

(a) To solve, use the node method.

KCL at $e_1$: $\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{8}\right)e_1 - \frac{1}{8}(8) - \frac{1}{4}(-12) = 0$

Solve for $e_1$:

$$\frac{5}{8}e_1 - 1 + 3 = 0 \Rightarrow \frac{5}{8}e_1 = -2$$

$$\Rightarrow e_1 = \frac{-16}{5} = -3.2 \text{ volts}$$

Therefore,

$$V_{8\Omega} = 8 - (-3.2) = 11.2 \text{ volts}$$

Unified Quiz 3S
November 7, 2003

NAME SOLUTIONS

**PROBLEM #2 (continued)**

$$i_{8\Omega} = \frac{V_{8\Omega}}{8\Omega} = 1.4\,A \quad \underline{\text{to}}\ \underline{\text{the}}\ \underline{\text{left}}$$

(b) From (a),

Potential at $P = -3.2\,V$

Potential at $Q = 0\,V$

$$\text{Potential difference} = 0 - (-3.2)$$

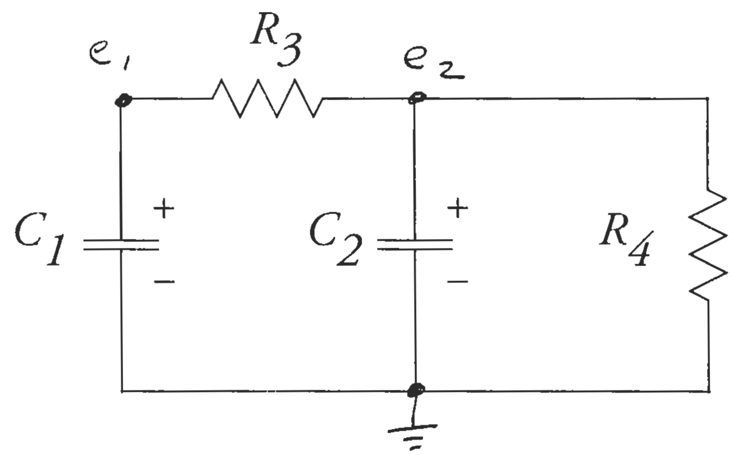
$$= 3.2$$

Q is at the higher potential

Unified Quiz 3S
November 7, 2003

NAME SOLUTIONS

**PROBLEM #3 (30%)**




For the circuit above, find $v_1(t)$ and $v_2(t)$, for the initial conditions

$$v_1(0) = 3 \text{ V}, \quad v_2(0) = 0 \text{ V}$$

The component values are

$$C_1 = 0.1 \text{ F}, \quad C_2 = 0.2 \text{ F}, \quad R_3 = 4 \ \Omega, \quad R_4 = 2 \ \Omega$$

Apply the node method, with node labellings shown.
Apply KCL at each node:

$$e_1 : \left(C_1 \frac{d}{dt} + G_3\right) e_1 \qquad - G_3 e_2 = 0$$

$$- G_3 e_1 + \left(C_2 \frac{d}{dt} + G_3 + G_4\right) e_2 = 0$$

Assume a solution of the form

$$e_1(t) = E_1 e^{st}, \quad e_2 = E_2 e^{st}$$

Then $E_1$ + $E_2$ satisfy

Unified Quiz 3S
November 7, 2003

NAME SOLUTIONS

**PROBLEM #3 (continued)**

$$\underbrace{\begin{bmatrix} C_1 s + G_3 & -G_3 \\ -G_3 & C_2 s + G_3 + G_4 \end{bmatrix}}_{M(s)} \underbrace{\begin{bmatrix} E_1 \\ E_2 \end{bmatrix}}_{\underline{E}} = 0$$

Plugging in component values,

$$M(s) = \begin{bmatrix} 0.1 s + 0.25 & -0.25 \\ -0.25 & 0.2 s + 0.75 \end{bmatrix}$$

To have a nontrivial solution,

$$\det(M(s)) = 0$$

$$= 0.02 s^2 + 0.125 s + 0.125$$

Solve by quadratic formula:

$$s = \frac{-0.125 \pm \sqrt{0.125^2 - 4(0.02)(0.125)}}{2(0.02)}$$

$$= -3.125 \pm 1.875$$

$$= -5, \ -1.25 \ \text{sec}^{-1}$$

For each characteristic value, solve for characteristic vector.

Unified Quiz 3S
November 7, 2003

NAME SOLUTIONS

**PROBLEM #3 (continued)**

$s_1 = -5$: $\quad M(-5) = \begin{bmatrix} 0.25 & 0.25 \\ 0.25 & 0.25 \end{bmatrix}$

$$\Rightarrow \underline{E}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \text{(or any multiple)}$$

$s_2 = -1.25$: $\quad M(-1.25) = \begin{bmatrix} 0.125 & -0.25 \\ -0.25 & 0.5 \end{bmatrix}$

Row reduce:

$$\begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \underline{E}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad \text{(or any multiple)}$$

Solution is:

$$\underline{e}(t) = a \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-5t} + b \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-1.25t}$$

The initial conditions are

$$\underline{e}(0) = a \begin{bmatrix} 1 \\ -1 \end{bmatrix} + b \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

**Unified Quiz 3S**
**November 7, 2003**

**NAME** SOLUTIONS

**PROBLEM #3 (continued)**

Solve by row reduction, Cramer's rule, or elimination of variables. The result is

$$a = 1$$

$$b = 1$$

Therefore,

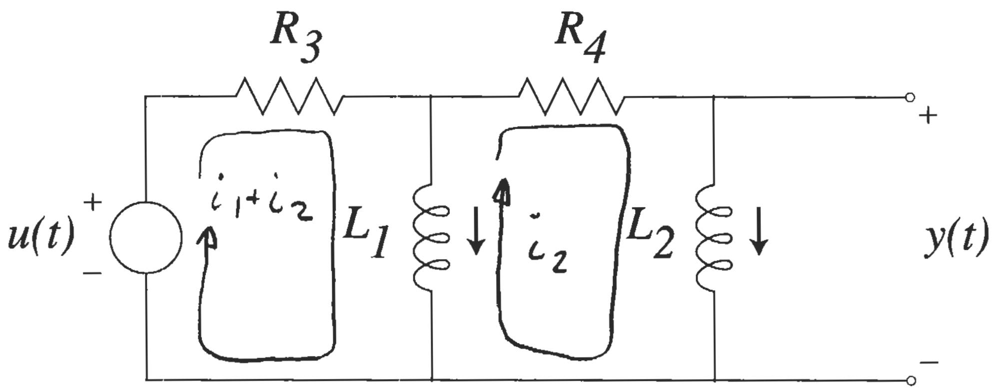
$$v_1(t) = e_1(t) = \left(e^{-5t} + 2e^{-1.25t}\right) \text{ volt}$$

$$v_2(t) = e_2(t) = \left(-e^{-5t} + e^{-1.25t}\right) \text{ volt}$$

Unified Quiz 3S
November 7, 2003

NAME SOLUTIONS

**PROBLEM #4 (25%)**




Find the differential equations that describe the input-output behavior of the circuit above, in state-space form,

$$\dot{\underline{x}}(t) = A\underline{x}(t) + Bu(t)$$
$$y(t) = C\underline{x}(t) + Du(t)$$

The component values are

$$L_1 = 1\ \text{H},\ \ L_2 = 2\ \text{H},\ \ R_3 = 3\ \Omega,\ \ R_4 = 5\ \Omega$$

States are $i_1, i_2$ (current through inductors).
State vector is

$$\underline{x} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

We must find $\dot{\underline{x}}$. Note that

$$\frac{di_1}{dt} = \frac{1}{L_1} v_1, \quad \frac{di_2}{dt} = \frac{1}{L_2} v_2$$

So need to find $v_1, v_2$ in terms of $i_1, i_2$, $u$. To do this, use the loop method. Note that

NAME SOLUTIONS

**PROBLEM #4 (continued)**

there are 2 loops, and 2 inductors, so loop currents are known, as shown. So apply KVL around Loops:

$i_1 + i_2$ loop: $R_3(i_1+i_2) + v_1 - u = 0$

$i_2$ loop: $R_4 i_2 + v_2 - v_1 = 0$

$$\Rightarrow \quad v_1 = -R_3(i_1+i_2) + u$$

$$v_2 = -R_4 i_2 + v_1$$

$$= -R_3 i_1 - (R_3+R_4) i_2 + u$$

Therefore,

$$\dot{\underline{x}} = \frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1/L_1 \\ v_2/L_2 \end{bmatrix}$$

$$= \begin{bmatrix} -R_3/L_1 & -R_3/L_1 \\ -R_3/L_2 & -(R_3+R_4)/L_2 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} 1/L_1 \\ 1/L_2 \end{bmatrix} u$$

$$= \underbrace{\begin{bmatrix} -3 & -3 \\ -1.5 & -4 \end{bmatrix}}_{A} \underline{x} + \underbrace{\begin{bmatrix} 1 \\ 1/2 \end{bmatrix}}_{B} u$$

Also,

$$y = v_2 = \begin{bmatrix} -R_3 & -(R_3+R_4) \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \end{bmatrix} u$$

**Unified Quiz 3S**
**November 7, 2003**

**NAME** Solutions

**PROBLEM #4 (continued)**

Therefore,

$$y = \underbrace{\begin{bmatrix} -3 & -8 \end{bmatrix}}_{C} \underline{x} + \underbrace{\begin{bmatrix} 1 \end{bmatrix}}_{D} u$$