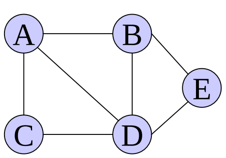
Introduction to Computers and Programming

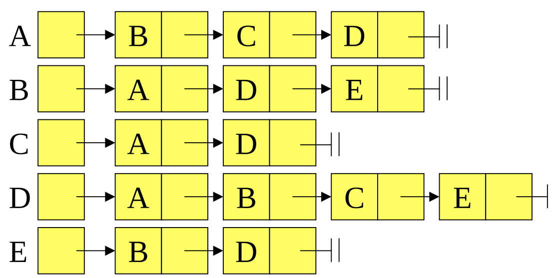
Prof. I. K. Lundqvist

Lecture 8
April 5 2004

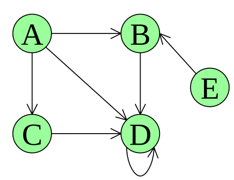
Today – More about Trees

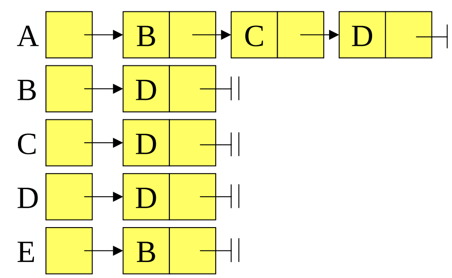
- Spanning trees
  - Prim's algorithm
  - Kruskal's algorithm
- Generic search algorithm
  - Depth-first search example
  - Handling cycles
  - Breadth-first search example





	A	B	C	D	E
A	0	1	1	1	0
B	1	0	0	1	1
C	1	0	0	1	0
D	1	1	1	0	1
E	0	1	0	1	0





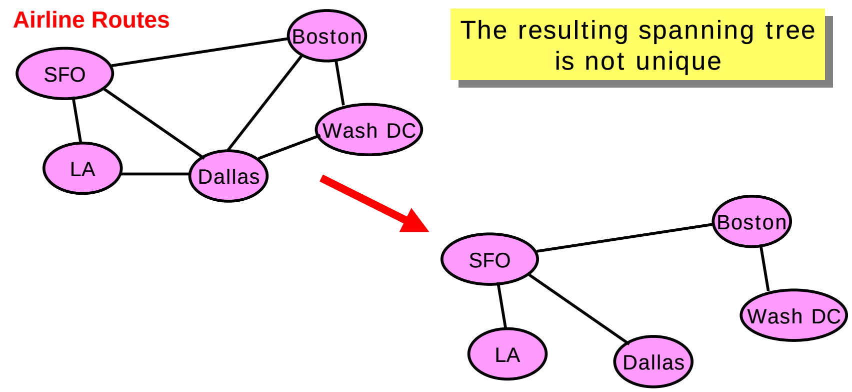
	A	B	C	D	E
A	0	1	1	1	0
B	0	0	0	1	0
C	0	0	0	1	0
D	0	0	0	1	0
E	0	1	0	0	0

Trees

- A tree is a connected graph without cycles
- A connected graph is a tree iff it has N vertices and N-1 edges
- A graph is a tree iff there is one and only one path joining any two of its vertices

Spanning Trees

- A Spanning tree of a graph G, is a tree that includes **all** the vertices from G.



Minimum Spanning Tree

- Prim's Algorithm
  - Finds a subset of the edges (that form a tree) including every vertex and the total weight of all the edges in tree is minimized
    - Choose starting vertex
    - Create the Fringe Set

    Initialization

    - Loop until the MST contains all the vertices in the graph
      - Remove edge with minimum weight from Fringe Set
      - Add the edge to MST
      - Update the Fringe Set

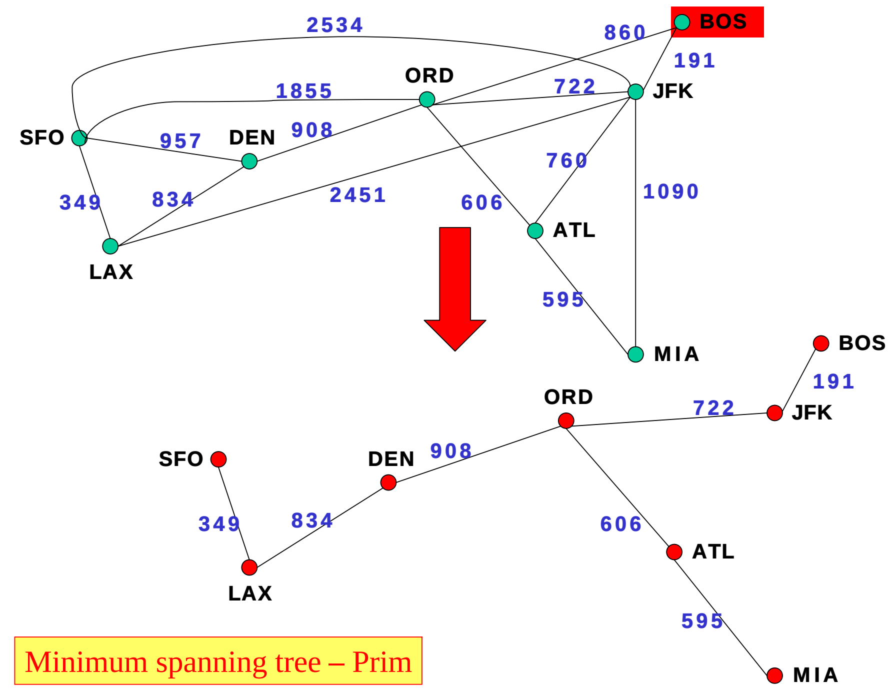
    Body

Prim – Initialization

- Pick any vertex x as the starting vertex
- Place x in the Minimum Spanning Tree (MST)
- For each vertex y in the graph that is adjacent to x
  - Add y to the Fringe Set
- For each vertex y in the Fringe Set
  - Set weight of y to weight of the edge connecting y to x
  - Set x to be parent of y

Prim – Body

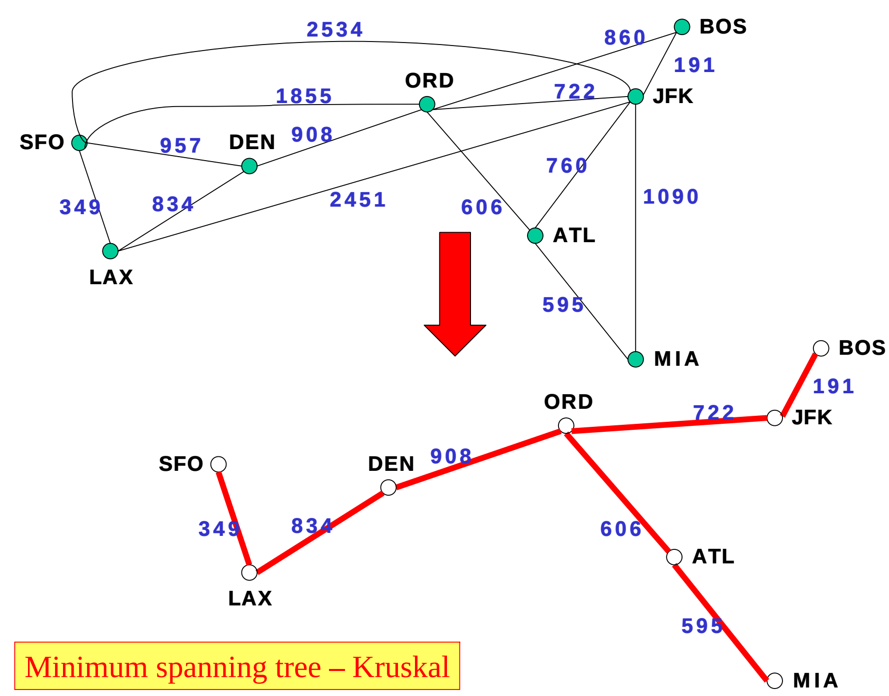
While number of vertices in MST < vertices in the graph
- Find vertex *y* with minimum weight in the Fringe Set
- Add vertex and the edge *x*,*y* to the MST
- Remove *y* from the Fringe Set
- For all vertices *z* adjacent to *y*
  - If *z* is not in the Fringe Set
    - Add *z* to the Fringe Set
    - Set parent to *y*
    - Set weight of *z* to weight of the edge connecting *z* to *y*
  - Else
    - If Weight(*y*,*z*) < Weight(*z*) then
      - Set parent to *y*
      - Set weight of *z* to weight of the edge connecting *z* to *y*



Minimum spanning tree – Prim

Minimum Spanning Tree

- Kruskal's Algorithm
  - Finds a minimum spanning tree for a connected weighted graph
    - Create a set of trees, where each vertex in the graph is a separate tree
    - Create set S containing all edges in the graph
    - While S not empty
      - Remove edge with minimum weight from S
      - if that edge connects two different trees, then add it to the forest, combining two trees into a single tree
      - Otherwise discard that edge



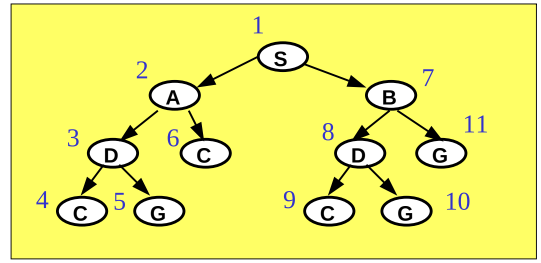
Minimum spanning tree – Kruskal

More about Trees

- Spanning trees
  - Prim's algorithm
  - Kruskal's algorithm
- Generic search algorithm
  - Depth-first search example
  - Handling cycles
  - Breadth-first search example

Depth First Search (DFS)

Idea:
•Explore descendants before siblings
•Explore siblings left to right



Where do we place the children on the queue?
- Assume we pick first element of Q
- Add path extensions to ? of Q

Simple Search Algorithm

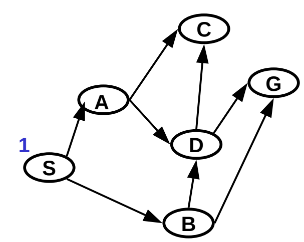
Let Q be a list of partial paths,
Let S be the start node and
Let G be the Goal node.

1. Initialize Q with partial path (S)
2. If Q is empty, fail. Else, pick a partial path N from Q
3. If head(N) = G, return N (goal reached!)
4. Else:
   a) Remove N from Q
   b) Find all children of head(N) and create all the one-step extensions of N to each child.
   c) Add all extended paths to Q
   d) Go to step 2.

Depth-First

Pick first element of Q; Add path extensions to front of Q

	Q
1	(S)
2	
3	
4	
5	



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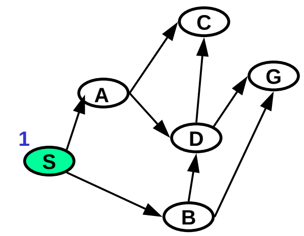
Depth-First

Pick first element of Q; Add path extensions to front of Q

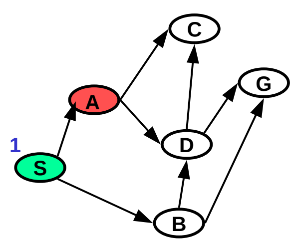
	Q
1	~~(S)~~
2	
3	
4	
5	



Depth-First

Pick first element of Q; Add path extensions to front of Q

	Q
1	~~(S)~~
2	(A S)
3	
4	
5	

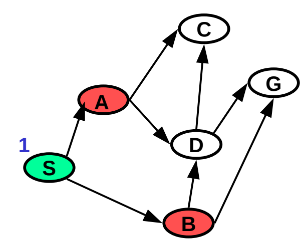


Added paths in blue

Depth-First

Pick first element of Q; Add path extensions to front of Q

	Q
1	~~(S)~~
2	(A S) (B S)
3	
4	
5	



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Simple Search Algorithm

Let Q be a list of partial paths,
Let S be the start node and
Let G be the Goal node.

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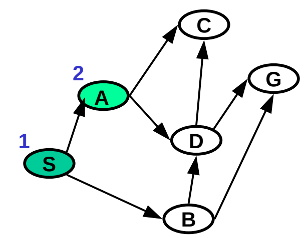
	Q
1	~~(S)~~
2	~~(A S)~~ (B S)
3	
4	
5	



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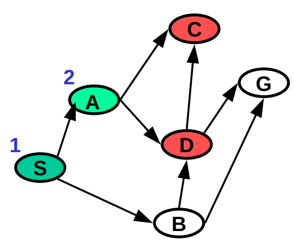
Depth-First

Pick first element of Q; Add path extensions to front of Q

	Q
1	~~(S)~~
2	~~(A S)~~ (B S)
3	(C A S) (D A S) (B S)
4	
5	



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Depth-First

Pick first element of Q; Add path extensions to front of Q

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1	~~(S)~~
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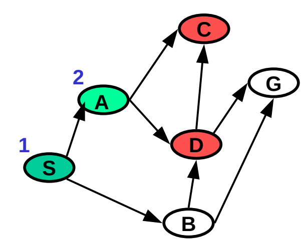
3	(C A S) (D A S) (B S)
4	
5	



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Depth-First

Pick first element of Q; Add path extensions to front of Q

	Q
1	(S)
2	(A S) (B S)
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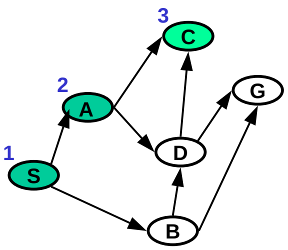
4	
5	



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Depth-First

Pick first element of Q; Add path extensions to front of Q

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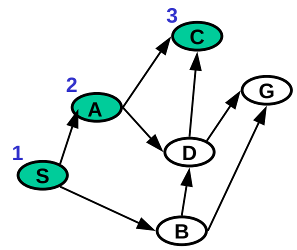

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Depth-First

Pick first element of Q; Add path extensions to front of Q

	Q
1	(S)
2	(A S) (B S)
3	(C A S) (D A S) (B S)
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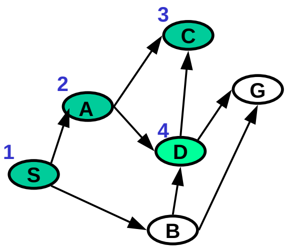
5	

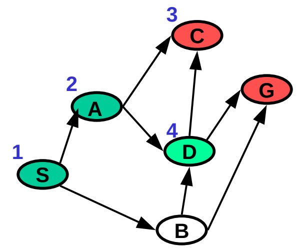


Added paths in blue

Depth-First

Pick first element of Q; Add path extensions to front of Q

	Q
1	(S)
2	(A S) (B S)
3	(C A S) (D A S) (B S)
4	(D A S) (B S)
5	(C D A S)(G D A S) (B S)



Simple Search Algorithm

Let Q be a list of partial paths,
Let S be the start node and
Let G be the Goal node.

1. Initialize Q with partial path (S)
2. If Q is empty, fail. Else, pick a partial path N from Q
3. If head(N) = G, return N (goal reached!)
4. Else:
   a) Remove N from Q
   b) Find all children of head(N) and
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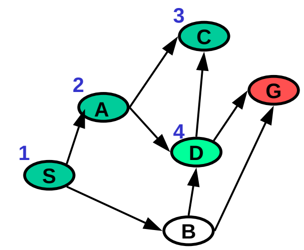
Depth-First

Pick first element of Q; Add path extensions to front of Q

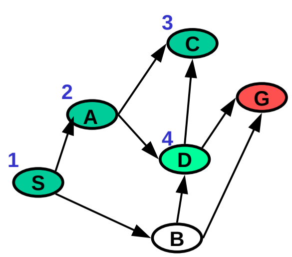
	Q
1	(S)
2	(A S) (B S)
3	(C A S) (D A S) (B S)
4	(D A S) (B S)
5	(C D A S)(G D A S) (B S)



Depth-First

Pick first element of Q; Add path extensions to front of Q

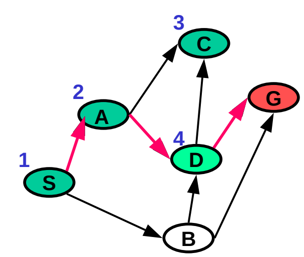
	Q
1	(S)
2	(A S) (B S)
3	(C A S) (D A S) (B S)
4	(D A S) (B S)
5	(C D A S)(G D A S) (B S)
6	(G D A S)(B S)



Depth-First

Pick first element of Q; Add path extensions to front of Q

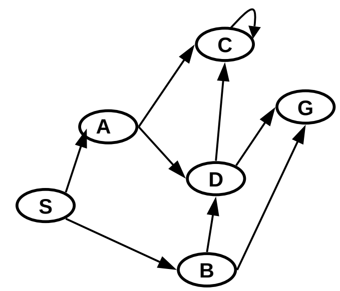
	Q
1	(S)
2	(A S) (B S)
3	(C A S) (D A S) (B S)
4	(D A S) (B S)
5	(C D A S)(G D A S) (B S)
6	(G D A S)(B S)



More about Trees

- Spanning trees
  - Prim's algorithm
  - Kruskal's algorithm
- Generic search algorithm
  - Depth-first search example
  - Handling cycles
  - Breadth-first search example

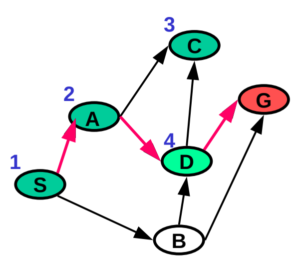
Issue: Starting at S and moving top to bottom, will depth-first search ever reach G?



Depth-First

Effort can be wasted in more mild cases

	Q
1	(S)
2	(A S) (B S)
3	(C A S) (D A S) (B S)
4	(D A S) (B S)
5	(C D A S)(G D A S) (B S)
6	(G D A S)(B S)



- C visited multiple times
- Multiple paths to C, D & G

How much wasted effort can be incurred in the worst case?

How Do We Avoid Repeat Visits?

Idea:

- Keep track of nodes already visited.
- Do not place visited nodes on Q.

Does this maintain correctness?

- Any goal reachable from a node that was visited a second time would be reachable from that node the first time.

Does it always improve efficiency?

- Guarantees each node appears at most once at the head of a path in Q.

Simple Search Algorithm

Let Q be a list of partial paths,
Let S be the start node and
Let G be the Goal node.

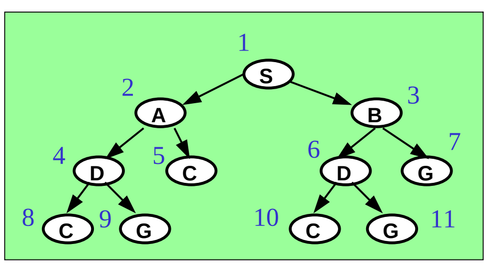
1. Initialize Q with partial path (S) as only entry; set Visited = ( )
2. If Q is empty, fail. Else, pick some partial path N from Q
3. If head(N) = G, return N (goal reached!)
4. Else
   a) Remove N from Q
   b) Find all children of head(N) not in Visited and create all the one-step extensions of N to each child.
   c) Add to Q all the extended paths;
   d) Add children of head(N) to Visited
   e) Go to step 2.

More about Trees

- Spanning trees
  - Prim's algorithm
  - Kruskal's algorithm
- Generic search algorithm
  - Depth-first search example
  - Handling cycles
  - Breadth-first search example

Breadth First Search (BFS)

Idea:
•Explore relatives at same level before their children
•Explore relatives left to right

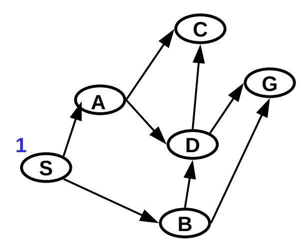


Where do we place the children on the queue?
- Assume we pick first element of Q
- Add path extensions to ? of Q

Breadth-First

Pick first element of Q; Add path extensions to end of Q

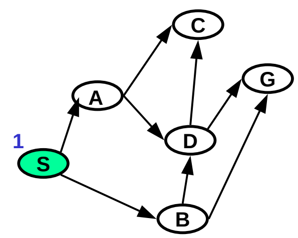
	Q	Visited
1	(S)	S
2		
3		
4		
5		
6		



Breadth-First

Pick first element of Q; Add path extensions to end of Q

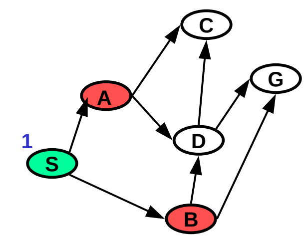
	Q	Visited
1	(S)	S
2		
3		
4		
5		
6		



Breadth-First

Pick first element of Q; Add path extensions to end of Q

	Q	Visited
1	(S)	S
2	(A S) (B S)	A,B,S
3		
4		
5		
6		



Breadth-First

Pick first element of Q; Add path extensions to end of Q

	Q	Visited
1	(S)	S
2	(A S) (B S)	A,B,S
3		
4		
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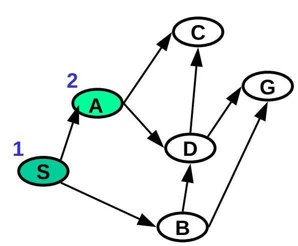
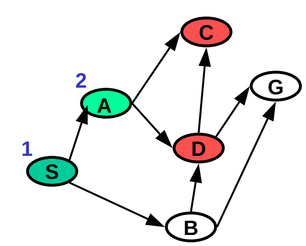


Breadth-First

Pick first element of Q; Add path extensions to end of Q

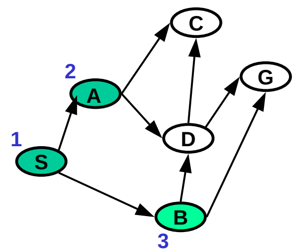
	Q	Visited
1	(S)	S
2	(A S) (B S)	A,B,S
3	(B S) (C A S) (D A S)	C,D,B,A,S
4		
5		
6		



Breadth-First

Pick first element of Q; Add path extensions to end of Q

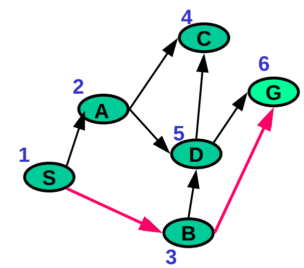
	Q	Visited
1	(S)	S
2	(A S) (B S)	A,B,S
3	(~~B S~~) (C A S) (D A S)	C,D,B,A,S
4		
5		
6		



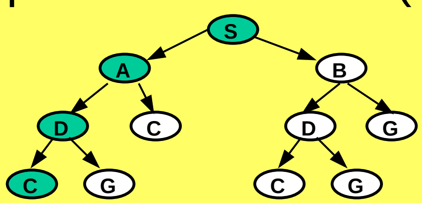
Breadth-First

Pick first element of Q; Add path extensions to end of Q

	Q	Visited
1	(S)	S
2	(A S) (B S)	A,B,S
3	(B S) (C A S) (D A S)	C,D,B,A,S
4	(C A S) (D A S) (G B S)*	G,C,D,B,A,S
5	(D A S) (G B S)	G,C,D,B,A,S
6	(G B S)	G,C,D,B,A,S



Depth First Search (DFS)

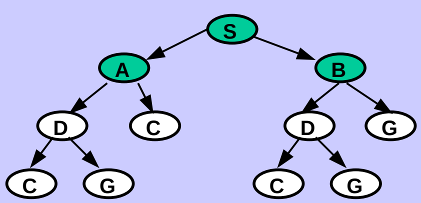


Depth-first:

Add path extensions to **front** of Q

Pick first element of Q

Breadth First Search (BFS)



Breadth-first:

Add path extensions to **back** of Q

Pick first element of Q

Test_ordered_binary.adb

Summary

- Most problem solving tasks may be formulated as state space search.
- Mathematical representations for search are graphs and search trees.
- Depth-first and breadth-first search may be framed, among others, as instances of a generic search strategy.
- Cycle detection is required to achieve efficiency and completeness.

- Document code
  - What it is doing
  - How it is doing it
  - What it is not doing (detailed status)
- Test run code
- Zip code