Introduction to Computers and Programming

Prof. I. K. Lundqvist

Some slides adapted from:
6.034 Tomas Lozano Perez,
Russell and Norvig AIMA and
16.410 Brian C. Williams

Lecture 6
Mar 30 2004

Today

- Problem Formulation
  – Problem solving as state space search
- Definition of Graphs
  – Types of Graphs
- Shortest Path problems
  – Dijkstra's Algorithm

Today

- **Problem Formulation**
  - **Problem solving as state space search**
- Definition of Graphs
  - Types of Graphs
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Complex missions must carefully:

- Plan complex sequences of actions
- Schedule tight resources
- Monitor and diagnose behavior
- Repair or reconfigure hardware.

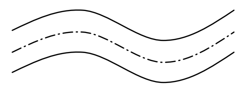
⇨ Most AI problems, like these, may be formulated as state space search.

Simple $\not\rightarrow$ Trivial

Can the astronaut get its produce safely across the Martian canal?

Astronaut
Goose
Grain
Fox

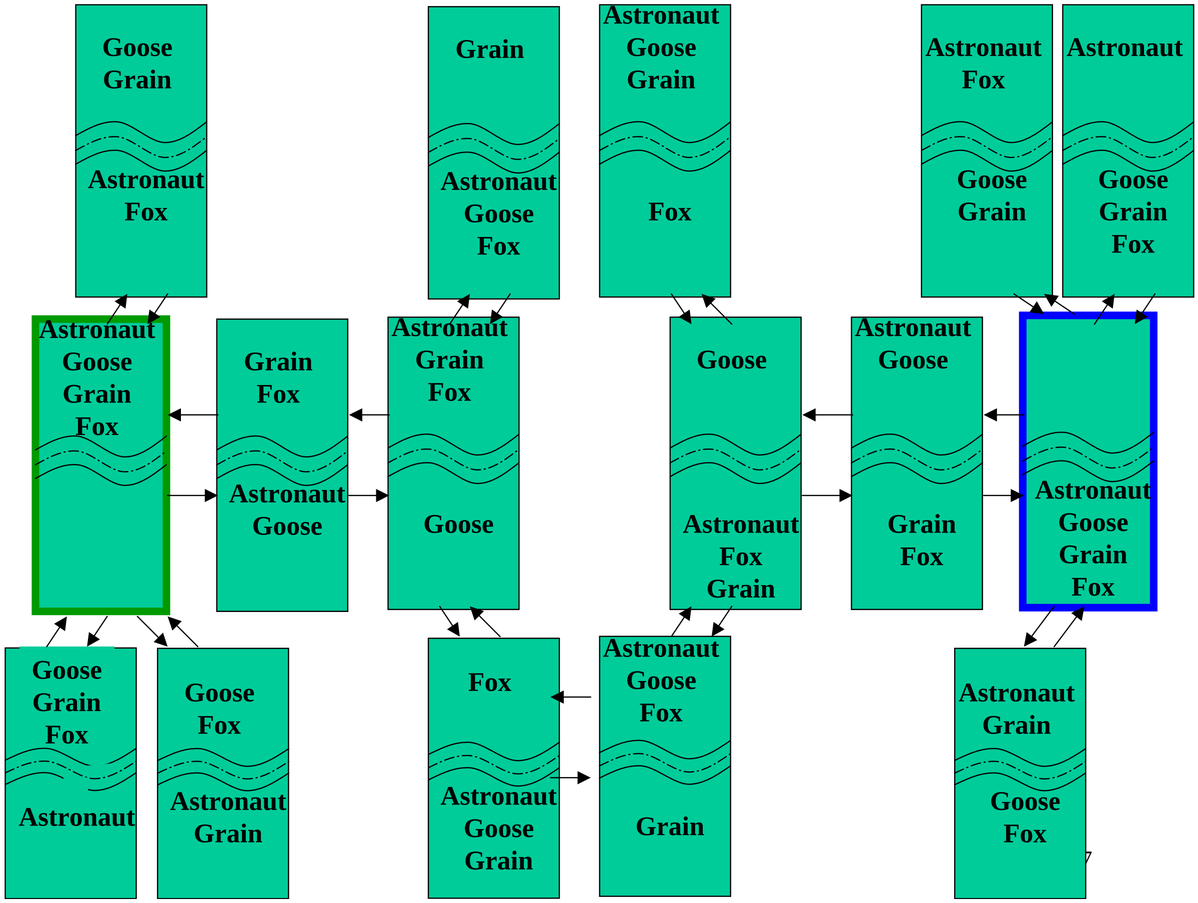
Rover



- Astronaut + 1 item allowed in the rover.
- Goose alone eats Grain
- Fox alone eats Goose

Problem Solving as State Space Search

- Formulate Goal
  - State
    - Astronaut, Fox, Goose & Grain across river
- Formulate Problem
  - States
    - Location of Astronaut, Fox, Goose & Grain at top or bottom river bank
  - Operators
    - Move rover with astronaut & 1 or 0 items to other bank
- Generate Solution
  - Sequence of States
    - Move(goose,astronaut), Move(astronaut), . . .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Astronaut Fox
Grain
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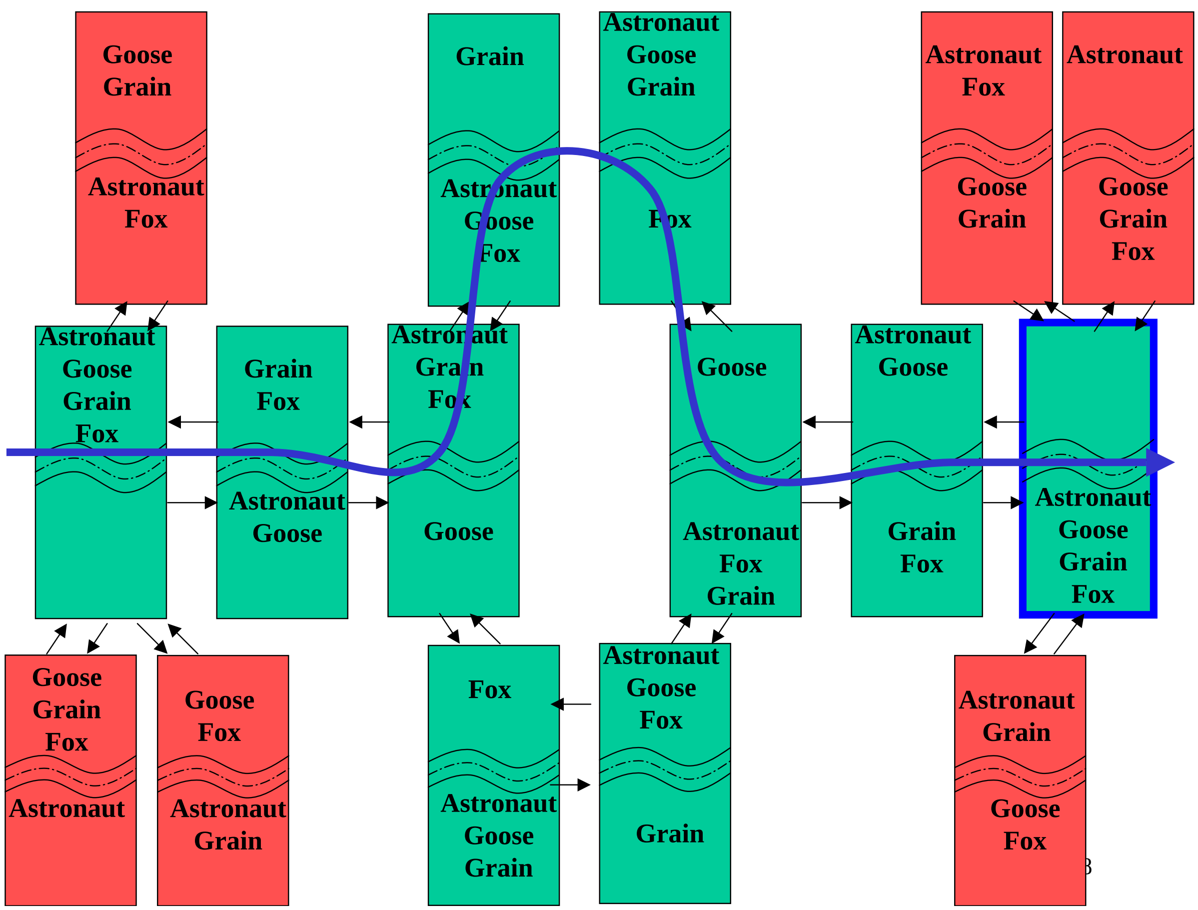
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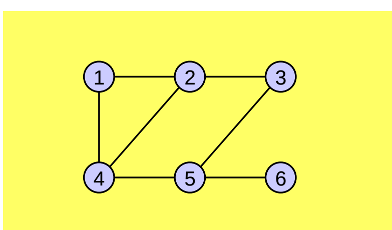
Today

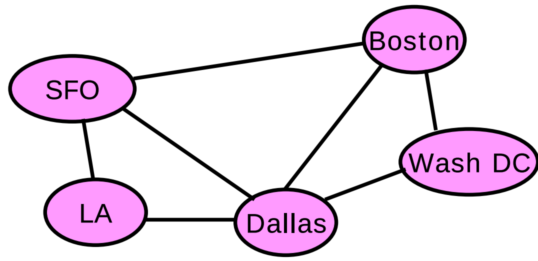
- Problem Formulation
  - Problem solving as state space search
- **Definition of Graphs**
  - **Types of Graphs**
- Shortest Path problems
  - Dijkstra's Algorithm

Graph

- A graph is a generalization of the simple concept of a set of dots (called **vertices** or nodes) connected by links (called **edges** or arcs)
  - Example: graph with 6 vertices and 7 edges



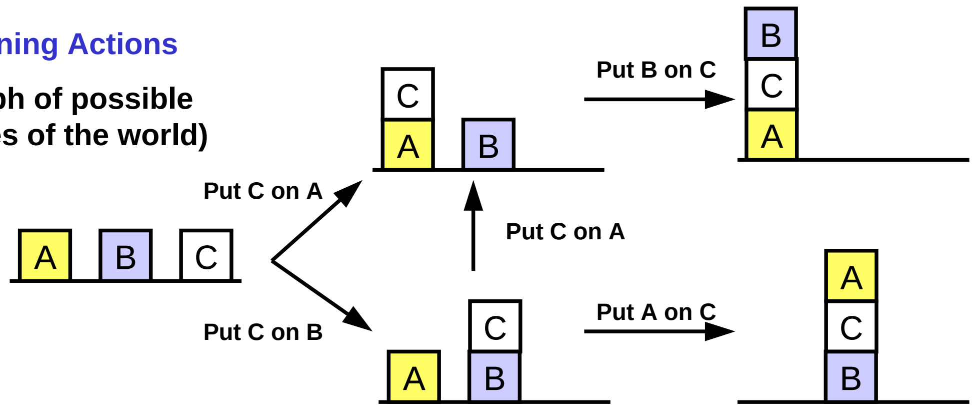
Examples of Graphs



Airline Routes

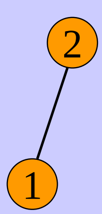
Planning Actions

(graph of possible states of the world)

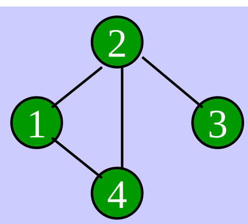


Graphs

- A **graph** G = (V, E) is a finite nonempty set of vertices and a set of edges

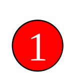


V = {1, 2, 3}
E = {(1, 2)}



V = {1, 2, 3, 4}
E = {(1,2)(2,3)(1,4)(2,4)}

- An **empty graph** is the graph whose edge set is empty

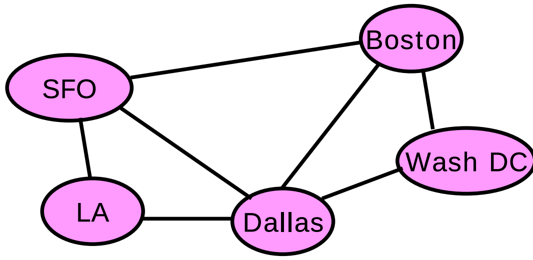


V = {1}
E = {∅}

- The **null graph** is the graph whose edge set and vertex set are empty

V = {∅}
E = {∅}

Examples of Graphs



Airline Routes

Graph AirlineRoutes is represented as the pair (V,E)

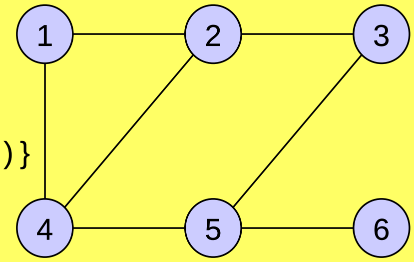
V= {Bos, SFO, LA, Dallas, Wash DC}

E= {(SFO,Bos),(SFO,LA),(LA,Dallas),(Dallas,Wash DC)...}

Graphs

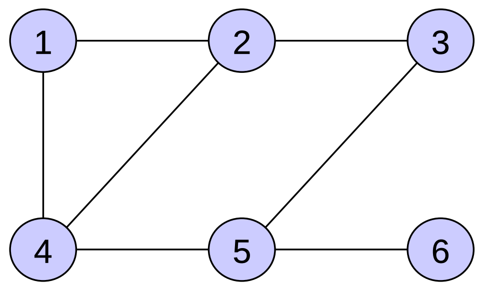
- A **loop** in a graph is an edge e in E whose endpoints are the same vertex.
- A **simple** graph is a graph with no loops, and there is at most one edge between any pair of vertices.

A simple graph with
V = {1, 2, 3, 4, 5, 6}
E = {(1,2), (1,4), (2,3), (2,4), (3,5), (5,6), (4,5)}

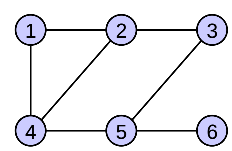


Graphs

- A **multigraph** has two or more edges that connect the same pair of vertices
- A **cycle** is a path that begins and ends with the same vertex
  - A cycle of length 1 is a loop
  - (1, 2, 3, 5, 4, 2, 1) is a cycle of length 6



Vertices

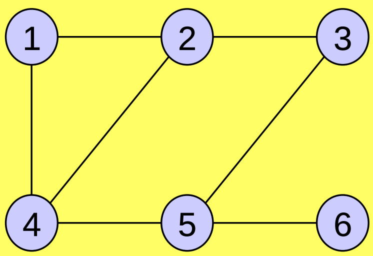


- Two vertices, *u* and *v* in an undirected graph G are called **adjacent** (or neighbors) in G, if {(*u*,*v*)} is an edge of G.

- The **degree** of a vertex in an undirected graph is the number of edges **incident** with it, except that a loop at a vertex contributes twice to the degree of that vertex.

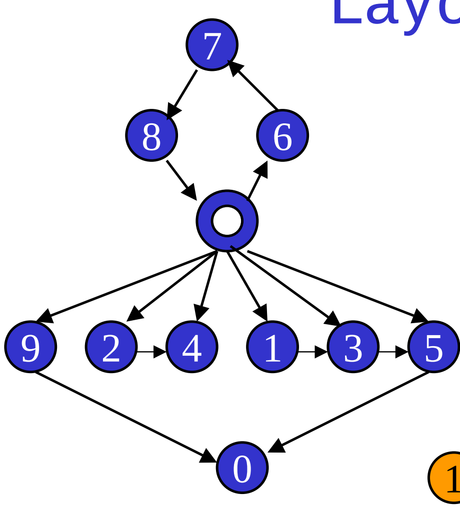
Adjacency Matrix

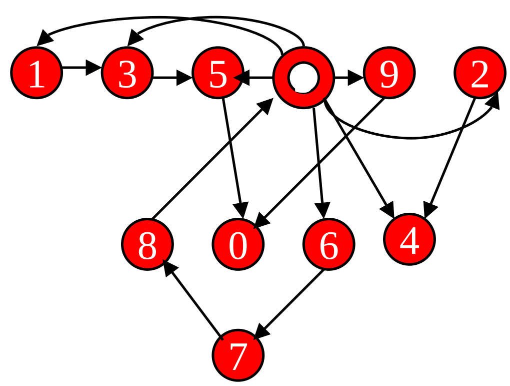
- A finite graph is often represented by its **adjacent matrix**.
  - An entry in row *I* and column *j* gives the number of edges from the i^{th} to the j^{th} vertex.

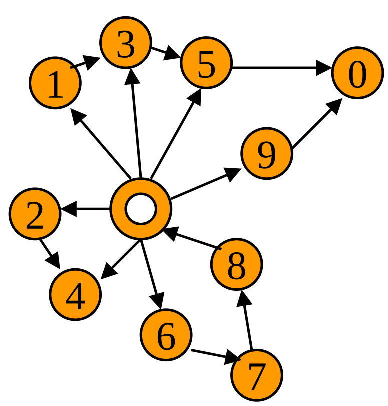


$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Layout of Graphs

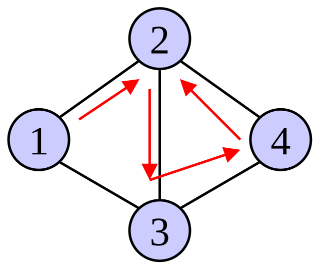




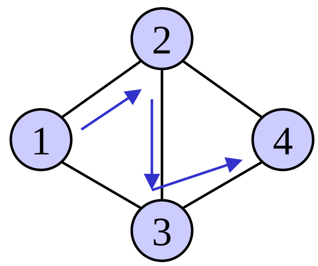


Walks and Paths

- A **walk** is a sequence of vertices (v_1, v_2, …, v_k) in which each *adjacent* vertex pair is an edge
- A **path** is a walk with no repeated vertices



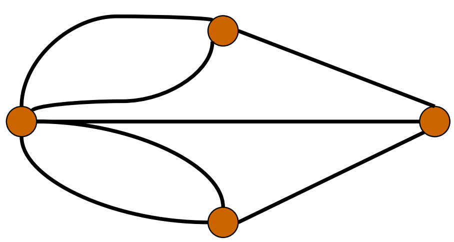
Walk (1,2,3,4,2)

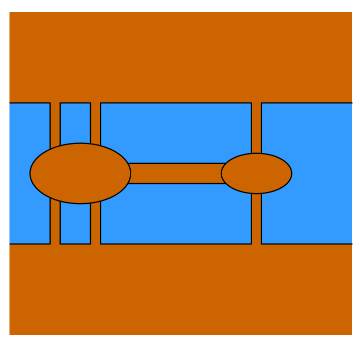


Path (1,2,3,4)

"The 1st problem in Graph Theory" Seven Bridges of Königsberg

- The city of Königsberg was set on the River Pregel, and included two large islands which were connected to each other and the mainland by seven bridges.
  - Was it possible to walk a route that crossed each bridge exactly once, and return to the starting point?



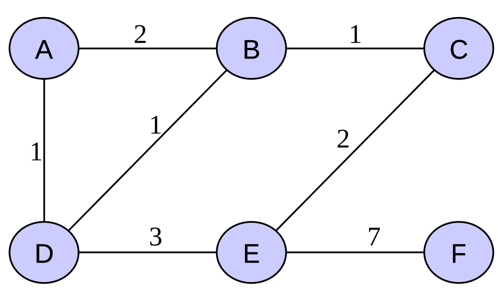


“The 1st problem in Graph Theory” Seven Bridges of Königsberg

- An **Eulerian path** in a graph is a path that uses each edge precisely once.
  - If such path exists, the graph is called traversable
- Euler showed that an Eulerian cycle exists if and only if all vertices in the graph are of even **degree**.

Weighted Graph

- A **weighted** graph associates a value (weight) to every edge in the graph.
  - A **weight of a path** in a weighted graph is the sum of the weights of the traversed edges.



- **Directed** graph (digraph) is a graph with one-way edges

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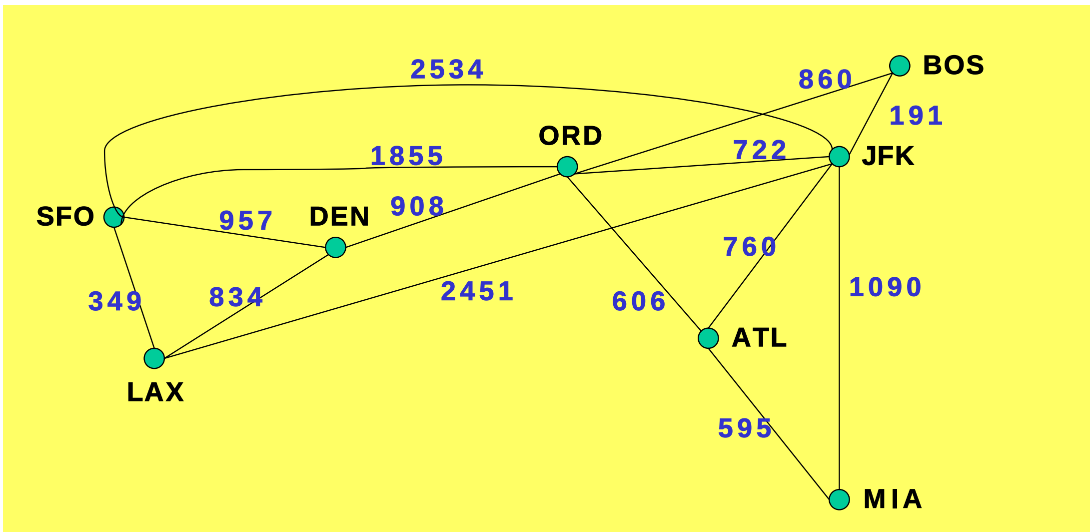
- **Shortest Path problems**
  - **Dijkstra's Algorithm**

Shortest Path Problems

- The **shortest path** from v_1 to v_2
  - Is the path of the smallest weight between the two vertices
  - Shortest may be least number of edges, least total weight, etc.
  - The weight of that path is called the **distance** between them

Shortest Path Problems

- Example: the weight can be mileage, fares, etc.



Shortest Path Problems

- **Dijkstra's algorithm**
  - Finds shortest path for a directed and connected graph G(V,E) which has non-negative weights.
  - Applications:
    - Internet routing
    - Road generation within a geographic region
    - ...

Dijkstra's Algorithm

- Dijkstra(G,w,s)

```
Init_Source(G,s)
S := empty set
Q := set of all vertices

while Q is not an empty set loop
    u := Extract_Min(Q)
    S := S union {u}
    for each vertex v which is a neighbor of u loop
        Relax(u,v,w)
```


Dijkstra's Algorithm

- Init_Source(G,s)

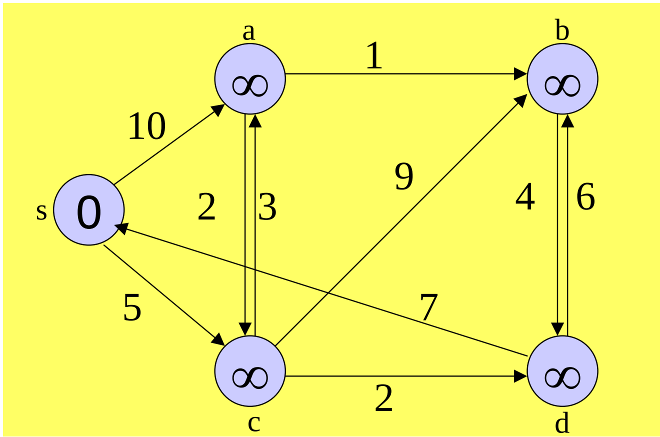
```
for each vertex v in V[G] loop
    d[v] := infinite
    previous[v] := 0
d[s] := 0
```

- v = Extract_Min(Q) searches for the vertex v in the vertex set Q that has the least d[v] value. That vertex is removed from the set Q and then returned.

- Relax(u,v,w)

```
if d[v] > d[u] + w(u,v) then
    d[v] := d[u] + w(u,v)
    previous[v] := u
```

Dijkstra’s Algorithm

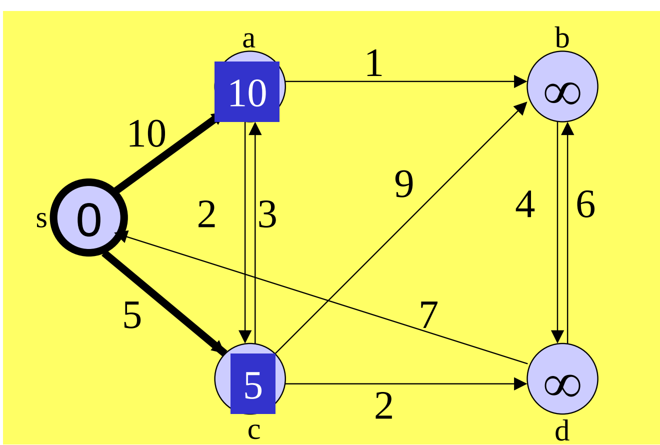


V = {a, b, c, d, s}
E = {(s,c), (c,d), (d,b), (b,d), (c,b), (a,c), (c,a), (a,b), (s,a)}

S = {∅}
Q = {s, a, b, c, d}

$$d = \begin{pmatrix} 0 \\ \infty \\ \infty \\ \infty \\ \infty \end{pmatrix} \qquad prev = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Dijkstra’s Algorithm



S = {s}
Q = {a, b, c, d}

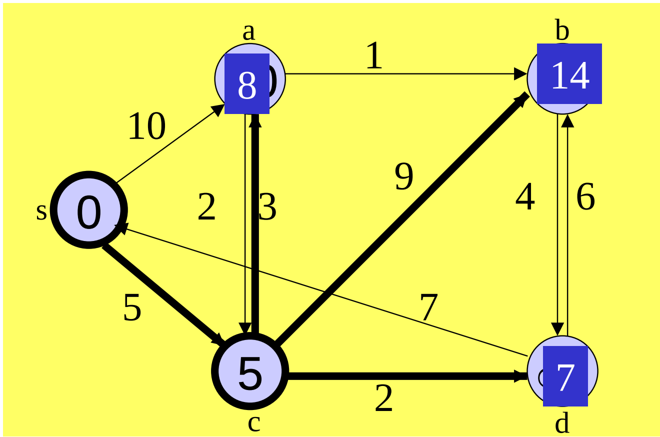
$$d = \begin{pmatrix} 0 \\ \infty \\ \infty \\ \infty \\ \infty \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 10 \\ \infty \\ 5 \\ \infty \end{pmatrix}$$

Extract_Min (Q) → s
Neighbors of s = a, c

Relax (s,c,5)
Relax (s,a,10)

$$prev = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ s \\ 0 \\ s \\ 0 \end{pmatrix}$$

Dijkstra's Algorithm



S = {s, c}
Q = {a, b, d}

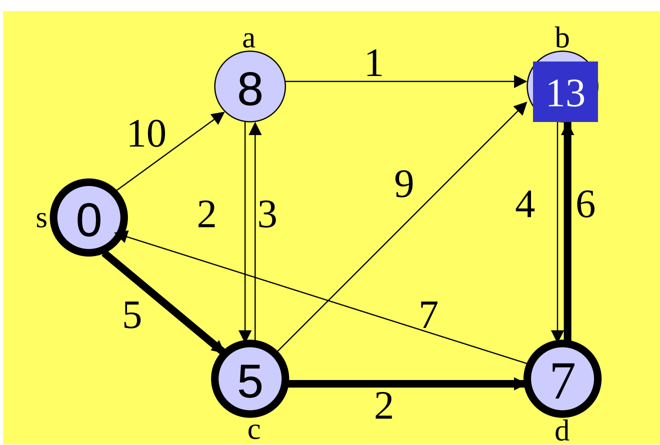
d = (0, 10, ∞, 5, ∞) → (0, 8, 14, 5, 7)

Extract_Min (Q) → c
Neighbors of c = a, b, d

prev = (0, s, 0, s, 0) → (0, c, c, s, c)

Relax (c,a,3)
Relax (c,b,9)
Relax (c,d,2)

Dijkstra's Algorithm



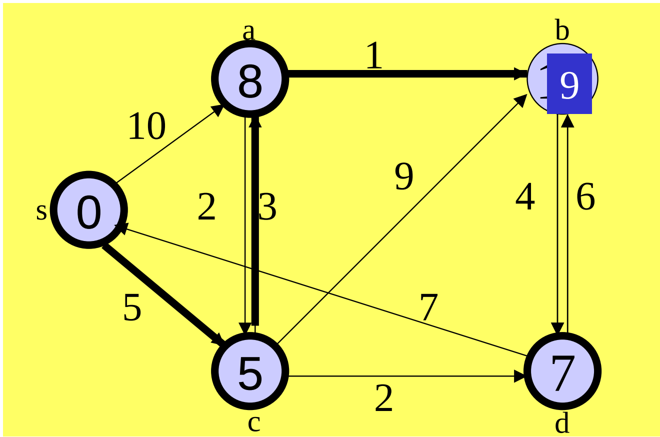
S = {s, c, d}
Q = {a, b}

d = (0, 8, 14, 5, 7) → (0, 8, 13, 5, 7)

Extract_Min (Q) → d
Neighbors of d = b
Relax (d,b,6)

prev = (0, c, c, s, c) → (0, c, d, s, c)

Dijkstra's Algorithm



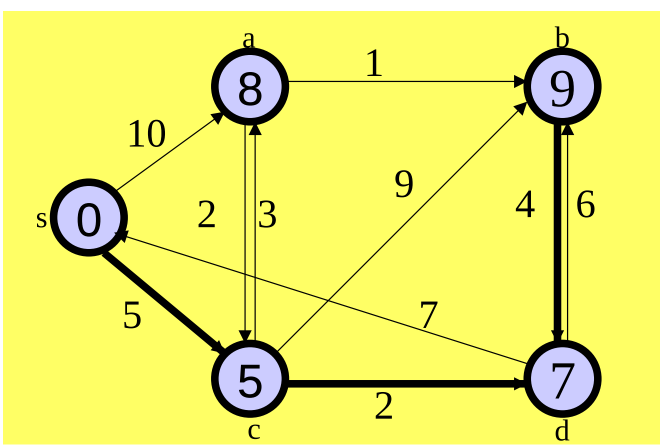
S = {s, c, d, a}
Q = {b}

$$d = \begin{pmatrix} 0 \\ 8 \\ 13 \\ 5 \\ 7 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 8 \\ 9 \\ 5 \\ 7 \end{pmatrix}$$

Extract_Min (Q) → a
Neighbors of a = b, c
Relax (a,b,1)
Relax (a,c,3)

$$prev = \begin{pmatrix} 0 \\ c \\ d \\ s \\ c \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ c \\ a \\ s \\ c \end{pmatrix}$$

Dijkstra's Algorithm



S = {s, c, d, a, b}
Q = {}

$$d = \begin{pmatrix} 0 \\ 8 \\ 9 \\ 5 \\ 7 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 8 \\ 9 \\ 5 \\ 7 \end{pmatrix}$$

Extract_Min (Q) → b
Neighbors of b = d
Relax (b, d, 4)

$$prev = \begin{pmatrix} 0 \\ c \\ a \\ s \\ c \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ c \\ a \\ s \\ c \end{pmatrix}$$