Introduction to Computers and Programming

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Lecture 22
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Elementary Logic

- Proposition is sentence that can be **either** true or false, not both
- Symbolic notations for manipulating logic of propositions
  - $\neg$ "not" or negation
  - $\wedge$ "and"
  - $\vee$ "or"
  - $\leftrightarrow$ "if and only if"
  - $\rightarrow$ "implies"
- Quantifiers
  - $\forall x p(x)$ "is true if for all x in U, p(x) is true"
  - $\exists x p(x)$ "is true if there exists an x such that p(x) is true"

Elementary Logic

- The proposition $q \rightarrow p$ is called the converse of $p \rightarrow q$, and $\neg q \rightarrow \neg p$ is the contrapositive of $p \rightarrow q$
- **Example:**
  - Give the converse of the following propositions
    - $q \rightarrow r$
    - If I am smart, then I am rich
    - If $x^2 = x$, then $x = 0$ or $x = 1$
    - If $2 + 2 = 4$, then $2 + 4 = 8$
  - Give the contrapositives for the propositions above

- Breaking assertions into component propositions
  - look for the logical operators!

Example:

If I go to Harry's or go to the country I will not go shopping.

P: I go to Harry's
Q: I go to the country
R: I will go shopping
If......P......or.....Q.....then....not.....R

$$(P \vee Q) \rightarrow \neg R$$

Elementary Logic

- Let p, q, r be the following propositions
  - p = “it is raining”
  - q = “the sun is shining”
  - r = “there are clouds in the sky”
- Translate the following into logical notation, using p, q, r and logical connectives
  - It is raining and the sun is shining
  - If it is raining, then there are clouds in the sky
  - If it is not raining, then the sun is not shining and there are clouds in the sky
  - The sun is shining if and only if it is not raining
  - If there are no clouds in the sky, then the sun is shining

Convert the following into predicate logic sentences

- Shamu can do every trick
- Shamu can do any trick
- Shamu cannot do every trick
- If any whale can do a trick, Shamu can
- If every whale can do a trick, Shamu can
- If any whale can do a trick, any whale can do a trick

$T\alpha$ = α is a trick
$W\alpha$ = α is a whale
$S\alpha$ = Shamu can do α
$C\alpha$ = α can do a trick
s = Shamu

$\forall x(Tx \rightarrow Sx)$ [Shamu can do every trick]
$\neg\forall x(Tx \rightarrow Sx)$ [Shamu cannot do every trick]

[If any whale can do a trick, Shamu can]

$\forall x$(Wx and Cx $\rightarrow$ Cs)

[If every whale can do a trick, Shamu can]
$\forall x(Wx \rightarrow Cx) \rightarrow Cs$

Proof by Cases

- Consider several cases that are *exhaustive*—i.e., that include all the possibilities
- **Example**: Prove that n^2-2 is not dividable by 5 for any positive integer
  - Case 1: n=5k
  - Case 2: n=5k+1
  - Case 3: n=5k+2
  - Case 4: n=5k+3
  - Case 5: n=5k+4

Proof by Contradiction

- We show that a conclusion holds by assuming it does not. If this leads to 'nonsense' contrary to reality (or hypotheses), then we have reached a contradiction.
- **Example**: Prove that there are infinitely many primes.
- **Example**: Prove that the sum of a rational number and an irrational number is always irrational.

Direct Proof

- Show that a given statement is true by simple combination of existing theorems with/without some mathematical manipulations
  - $H_1 \wedge H_2 \wedge \ldots \wedge H_n \Rightarrow C$
- Proof of the contrapositive (indirect proof)
  - $\neg C \Rightarrow \neg(H_1 \wedge H_2 \wedge \ldots \wedge H_n)$

Direct or Indirect Proof?

- **Example**: Let m, n $\in \mathbb{N}$. Prove that if m+n >= 73 then m >= 37 or n >= 37

Proof by Contradiction

- $(H_1 \wedge H_2 \wedge \ldots \wedge H_n) \wedge \neg C \Rightarrow$ a contradiction
- **Example**: If 5n+6 is odd, then n is odd
- **Example**: prove that at least 4 of any 22 days must fall on the same day of the week

Logic in proofs

Rule of Inference	Tautology	Name
p $\therefore p \vee q$	$p \rightarrow (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \rightarrow p$	Simplification
p, q $\therefore p \wedge q$	$(p \wedge q) \rightarrow p \wedge q$	Conjunction
$p, p \rightarrow q$ $\therefore q$	$(p \wedge (p \rightarrow q)) \rightarrow q$	Modus Ponens
$\neg q, p \rightarrow q$ $\therefore \neg p$	$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$	Modus Tollens
$p \rightarrow q, q \rightarrow r$ $\therefore p \rightarrow r$	$((p \rightarrow q) \wedge (q \rightarrow r))$ $\rightarrow (p \rightarrow r)$	Hypothetical Syllogism
$p \vee q, \neg p$ $\therefore q$	$((p \vee q) \wedge \neg p) \rightarrow q$	Disjunctive Syllogism
$p \vee q, \neg p \vee r$ $\therefore q \vee r$	$(p \vee q) \wedge (\neg p \vee r)$ $\rightarrow q \vee r$	Resolution

- **Example**: Convert each of the following arguments into logical notation using the suggested variables. Then provide a formal proof.
  - "if my computations are correct and I pay the electric bill, then I will run out of money. If I do not pay the electric bill, the power will be turned off. Therefore, if I do not run out of money and the power is still on, then my computations are incorrect". (c, b, r, p)

- Let
  - c := "my computations are correct"
  - b := "I pay the electric bill"
  - r := "I run out of money"
  - p := "the power stays on"

- Then theorem is:
  - if $(c \wedge b) \rightarrow r$ and $\neg b \rightarrow \neg p$, then $(\neg r \wedge p) \rightarrow \neg c$

Mathematical Induction

- Let p(m), p(m+1), ..., p(n) be a sequence of propositions. If
  (B) p(m) is true, and
  (I) p(k+1) is true whenever p(k) is true and $m \le k < n$,
  then all propositions are true.
- **Example**: for each positive integer n, let p(n) be "**$n! > 2^n$**," a proposition that we claim is true for $n \ge 4$.
  To give a proof by induction, we verify that p(n) for n=4, and then show
  (I) If $4 \le k$ and $k! > 2^k$, then $(k+1)! > 2^{k+1}$

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Division Leader - Software Engineering

- **When**: Wednesday 5/12
- **Topic:** What Software Do You Need to Get to Mars? A Look at Large Scale Software Development
  - Today's aerospace applications are extremely software intensive. The designs of all modern flight platforms, e.g., aircraft, rotorcraft, submersibles, missiles and spacecraft, have been revolutionized because of the integration of advanced microelectronics and complex sensor systems. All of these systems extensively use information generated by a myriad of sensor subsystems that are highly integrated into a real-time embedded software architecture. This lecture will explore what it takes to develop large scale software for a number of examples of highly reliable, flight critical software applications.