Introduction to Computers and Programming

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Lecture 19
May 4 2004

Terminology

- A proposition is a declarative statement that is **either true** or **false** (but not both)
- **Conjunction**: $p \wedge q$, $\wedge$ corresponds to *and*
- **Disjunction**: $p \vee q$, $\vee$ corresponds to *or*
- **Negation**: $\neg p$, $\neg$ corresponds to *not*

CQ 1

P = Everyone loves ice cream; Q = X loves ice cream

P is a proposition, Q is a proposition

1. True, False
2. True, True
3. False, False
4. I don't know

Implication

- **Implication**: $p \rightarrow q$, $\rightarrow$ corresponds to *implies*

"if it rains, then it is cloudy"

p = it rains
q = it is cloudy
$p \rightarrow q$

P	Q	$\overline{P}$	$\overline{P} \vee Q$
0	0	1	1
0	1	1	1
1	0	0	0
1	1	0	1

Rule of Inference	Tautology	Name
p $\therefore p \vee q$	$p \to (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \to p$	Simplification
p, q $\therefore p \wedge q$	$(p \wedge q) \to p \wedge q$	Conjunction
$p, p \to q$ $\therefore q$	$(p \wedge (p \to q)) \to q$	Modus Ponens
$\neg q, p \to q$ $\therefore \neg p$	$(\neg q \wedge (p \to q)) \to \neg p$	Modus Tollens
$p \to q, q \to r$ $\therefore p \to r$	$((p \to q) \wedge (q \to r)) \to (p \to r)$	Hypothetical Syllogism
$p \vee q, \neg p$ $\therefore q$	$((p \vee q) \wedge \neg p) \to q$	Disjunctive Syllogism
$p \vee q, \neg p \vee r$ $\therefore q \vee r$	$(p \vee q) \wedge (\neg p \vee r) \to q \vee r$	Resolution

Ex: Consider these statements "If I buy something, then I go to the store." and "If I go to the store, then I drive my car." If these two statements are true, then by ***hypothetical syllogism*** we can conclude that "If I buy something, then I drive my car."

Ex: Consider the statements "It is raining today or it is snowing today." and "It is not snowing today or it is windy today." If we know both of these statements are true then **what can we conclude?**

By the ***rule of resolution***, we know that "It is raining today or it is windy today."

Ex: Show that

$$[(\neg p \wedge q) \wedge (r \rightarrow p) \wedge (\neg r \rightarrow s) \wedge (s \rightarrow t)] \rightarrow t$$

is a true statement.

Proof:

We assume the hypotheses $(\neg p \wedge q)$, $(r \rightarrow p)$, $(\neg r \rightarrow s)$, and $(s \rightarrow t)$

1. By $(\neg p \wedge q)$ we know $\neg p$ [simplification]
2. By $(r \rightarrow p)$ we know $\neg p \rightarrow \neg r$ [contrapositive]
3. By 2 and $(\neg r \rightarrow s)$ we know $(\neg p \rightarrow s)$ [hypothetical syllogism]
4. By 3 and $(s \rightarrow t)$ we know $(\neg p \rightarrow t)$ [hypothetical syllogism]
5. By 1 and 4 we know **t** [modus ponens]

Ex: **A detective has interviewed four witnesses to a crime**. From the stories of the witnesses the detective has concluded that if

- the butler is telling the truth, then so is the cook;
- the cook and the gardener can not both be telling the truth;
- the gardener and the handyman are not both lying;
- and if the handyman is telling the truth then the cook is lying.

For each of the four witnesses, can the detective determine whether that person is telling the truth or lying?

(1) $b \rightarrow c$		
(2) $\neg(c \wedge g)$	or	$\neg c \vee \neg g$
(3) $\neg(\neg g \wedge \neg h)$	or	$g \vee h$
(4) $h \rightarrow \neg c$		

(1) $b \to c$	or	$\neg b \vee c$
(2) $\neg(c \wedge g)$	or	$\neg c \vee \neg g$
(3) $\neg(\neg g \wedge \neg h)$	or	$g \vee h$
(4) $h \to \neg c$	or	$\neg h \vee \neg c$

By combining (1) and (2) we get	(5) $\neg b \vee \neg g$
By combining (1) and (4) we get	(6) $\neg b \vee \neg h$
By combining (2) and (3) we get	(7) $\neg c \vee h$
By combining (3) and (4) we get	(8) $g \vee \neg c$

(1) $\neg b \vee c$	(5) $\neg b \vee \neg g$
(2) $\neg c \vee \neg g$	(6) $\neg b \vee \neg h$
(3) $g \vee h$	(7) $\neg c \vee h$
(4) $\neg h \vee \neg c$	(8) $g \vee \neg c$

By combining (1) and (7) we get	(9) $\neg b \vee h$
By combining (1) and (8) we get	(10) $\neg b \vee g$
By combining (2) and (8) we get	(11) $\neg c \vee \neg c \equiv \neg c$
By combining (3) and (5) we get	(9) $\neg b \vee h$
By combining (3) and (6) we get	(10) $\neg b \vee g$
By combining (4) and (7) we get	(11) $\neg c \vee \neg c \equiv \neg c$
By combining (5) and (8) we get	(12) $\neg b \vee \neg c$
By combining (6) and (7) we get	(12) $\neg b \vee \neg c$

(1) $\neg b \vee c$	(5) $\neg b \vee \neg g$	(9) $\neg b \vee h$
(2) $\neg c \vee \neg g$	(6) $\neg b \vee \neg h$	(10) $\neg b \vee g$
(3) $g \vee h$	(7) $\neg c \vee h$	(11) $\neg c$
(4) $\neg h \vee \neg c$	(8) $g \vee \neg c$	(12) $\neg b \vee \neg c$

By combining (9) and (4) we get (12) $\neg b \vee \neg c$

By combining (9) and (6) we get (13) $\neg b \vee \neg b \equiv \neg b$

By combining (10) and (2) we get (12) $\neg b \vee \neg c$

By combining (10) and (5) we get (13) $\neg b \vee \neg b \equiv \neg b$

By combining (11) and (1) we get (13) $\neg b$

By combining (12) and (1) we get (13) $\neg b \vee \neg b \equiv \neg b$

We can see that (13) won't combine with anything so we're done.
We have come to the same conclusions as before $\neg b$ and $\neg c$.

Predicate Logic

- Predicate logic: composed of ***atomic sentences*** which are made up of:
  - **Constants**, or objects, such as "Butterflies"
  - **Variables**, such as X
  - **Predicate** such as "eats", "likes", "bigger than"

- It also uses ***quantifiers*** which allow general statements such as ***All*** *butterflies are colorful*, or ***There are some*** *elephants that like mangoes*

Universal Quantifiers

Let **P(x)** be a predicate on some universe of discourse.

The **universal quantifier** of **P(x)** is the proposition:
"P(x) is true for all x in the universe of discourse"

Formally: **$\forall$x P(x)** is read as "for all x, P(x)"

- $\forall$x P(x) is TRUE if P(x) is true for every single x
- $\forall$x P(x) is FALSE if there is an x for which P(x) is false

Finite Universes

($\forall$x Work_Hard(x) $\rightarrow$ get_an_A(x))

In the special case that the universe of discourse,
$U = \{x_1, x_2, x_3, \ldots, x_n\}$

$$\forall x\, P(x)$$

$$\Downarrow$$

$$P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n)$$

Existential Quantifiers

Let **P(x)** be a predicate on some universe of discourse U.

The **existential quantifier** of **P(x)** is the proposition:
"P(x) is true if it is true for at least one x in the universe of discourse"

Formally, **∃x P(x)**: is read as "for some x, P(x)"

- ∃x P(x) is FALSE if P(x) is false for every single x
- ∃x P(x) is TRUE if there is an x for which P(x) is true

Finite Universes

∃x x is awake

In the special case that the universe of discourse, U, is finite, (U = $\{x_1, x_2, x_3, \ldots, x_n\}$)

$$\exists x\ P(x)$$

$$\Downarrow$$

$$P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n)$$

Lions and Coffee

Given U = all creatures:

- **L(x)** = “x is a lion.”
- **F(x)** = “x is fierce.”
- **C(x)** = “x drinks coffee.”

All lions are fierce.

$$\forall x\ (L(x) \rightarrow F(x))$$

Some lions don’t drink coffee.

$$\exists x\ (L(x) \land \neg C(x))$$

Some fierce creatures don’t drink coffee.

$$\exists x\ (F(x) \land \neg C(x))$$

Butterflies and Nectar

B(x) = “x is a butterfly.”
L(x) = “x is a large butterfly.”
N(x) = “x lives on nectar.”
R(x) = “x is richly covered.”

All butterflies are richly colored.

$$\forall x\ (\mathbf{B(x)} \rightarrow R(x))$$

No large butterflies live on nectar.

$$\neg\exists x\ (L(x) \land N(x))$$

Insects that do not live on nectar are dully colored

CQ 2

Insects that do not live on nectar are dully colored

1. $\forall x\ (\neg N(x) \rightarrow \neg R(x))$

2. $\forall x\ (N(x) \rightarrow R(x))$

3. $\forall x\ (\neg N(x) \rightarrow R(x))$

4. Don’t know

Negating Quantifiers

- $\neg\forall$**x P(x)** is the same as $\exists$**x** $\neg$**P(x)**

- $\neg\exists$**x P(x)** is the same as $\forall$**x** $\neg$**P(x)**

Rule of Thumb: to negate a quantifier, move negation to the right, changing quantifiers as you go

Quantifier Negation

No large insects live on nectar

$\neg\exists x\ (L(x) \wedge N(x)) = \forall x\ \neg(L(x) \wedge N(x))$ Negation rule

$= \forall x\ (\neg L(x) \vee \neg N(x))$ DeMorgan's

$= \forall x\ (L(x) \rightarrow \neg N(x))$ Subst for $\rightarrow$

Proofs

- A ***theorem*** is a statement that can be shown to be true
- A ***proof*** is the means of doing so

Axioms, postulates, hypotheses, previously proven theorems

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Rules of Inference

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Proof