Introduction to Computers and Programming

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Lecture 18
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Four Variable K-Maps Example-2

Using a 4-variable K-Map, simplify the following Truth table

A	B	C	D	Output
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	**1**
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	**1**
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	**1**
1	1	1	1	0

CD \ AB	00	01	11	10
00	0	1	0	0
01	0	0	0	1
11	0	0	0	0
10	0	0	1	0

Output = $\overline{A}B\overline{C}\overline{D} + A\overline{B}\overline{C}D + ABC\overline{D}$

Four Variable K-Maps Example-2

Using a 4-variable K-Map, simplify the following Truth table

A	B	C	D	Output
0	0	0	0	**1**
0	0	0	1	**1**
0	0	1	0	0
0	0	1	1	0
0	1	0	0	**1**
0	1	0	1	**1**
0	1	1	0	0
0	1	1	1	0
1	0	0	0	**1**
1	0	0	1	0
1	0	1	0	**1**
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	**1**
1	1	1	1	**1**

CD \ AB	00	01	11	10
00				
01				
11				
10				

Product-of-Sums from a Truth Table

A	B	C	F	$\overline{F}$
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

Find an expression for $\overline{F}$

$$\overline{F} = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}B\overline{C}$$

$$F = \overline{\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}B\overline{C}}$$

$$F = \overline{\overline{A}\,\overline{B}\,\overline{C}} \bullet \overline{\overline{A}\,\overline{B}C} \bullet \overline{\overline{A}B\overline{C}}$$

$$F = (A+B+C) \bullet (A+B+\overline{C}) \bullet (A+\overline{B}+C)$$

Maxterms

A	B	C	F	$\overline{F}$
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

Maxterms

$$F = (A + B + C) \bullet (A + B + \overline{C}) \bullet (A + \overline{B} + C)$$

- To find a Product-of-Sums form for a truth table
  - Make one maxterm for each row in which the function is **zero**
  - For each maxterm, each variable appears once
    - In its **complemented** form if it is **one** in the row
    - In its **regular** form if it is **zero** in the row

Today

- Propositional Logic
- From English to propositions
- Quantified statements
- Tomorrow: Methods of proving theorems

Propositional Logic

- Logic at the sentential level
  - Smallest unit: sentence
  - Sentences that can be **either** true or false
  - This kind of sentences are called **Propositions**

- If a propositions is true, then its **truth value** is "true", if proposition if false, then the truth value is "false"

Propositional Logic

- The following are propositions:
  - Grass is green
  - 2 + 4 = 4

- The following are not propositions:
  - Wake up
  - Is it raining today?
  - X > 2
  - X = X

Elements of Propositional Logic

Connectives

not	$\neg$
and	$\wedge$
or	$\vee$
if_then (implies)	$\rightarrow$
iff	$\leftrightarrow$

Connectives

P	Q	$(P \vee Q)$
F	F	F
F	T	T
T	F	T
T	T	T

P	Q	$(P \wedge Q)$
F	F	F
F	T	F
T	F	F
T	T	T

P	Q	$(P \rightarrow Q)$
F	F	T
F	T	T
T	F	F
T	T	T

P	$\neg P$
T	F
F	T

P	Q	$(P \leftrightarrow Q)$
F	F	T
F	T	F
T	F	F
T	T	T

Truth tables

Concept Question

Given P → Q, Is Q → P True?

1. Yes
2. No
3. I don't know
4. What is P->Q

Converse and Contrapositive

- For P → Q

  Q → P is called its **converse**

  ¬Q → ¬P is called its **contrapositive**

Example: If it rains, then I get soaked

converse :
If I get soaked, then it rains

contrapositive :
If I don't get soaked, then it does not rain

From English to Proposition

- Premises:

P – It snows

Q – If it snows, then the school is closed

The school is closed

- Rules of inference

$[P \wedge (P \rightarrow Q)] \rightarrow Q$

From English to Proposition

- Restate given statements using building blocks and the connectives
- Propositions
  - P it is raining
  - Q I will go to the beach
  - R I have time
- "I will go to the beach if it is not raining" restate "If it is not raining, I will go to the beach" restate $\neg P \rightarrow Q$

Exercise

- Restate: " I will go to the beach if is not raining and I have time "

"If it is not raining and I have time, then I will go to the beach"

$(\neg P \wedge R) \rightarrow Q$

Rule of Inference: Modus Ponens

$(p \wedge (p \rightarrow q)) \rightarrow q$ is a **tautology**. It states that if we know that both an *implication* $p \rightarrow q$ is true and that its *hypothesis*, p, is true, then the *conclusion*, q, is true.

Ex: Suppose the *implication* "If the bus breaks down, then I will have to walk" and its hypothesis "the bus breaks down" are true.
Then by *modus ponens* it follows that "I will have to walk".

Ex: Assume that the *implication* $(n > 3) \rightarrow (n^2 > 9)$ is true.
Suppose also that $n > 3$.
Then by *modus ponens*, it follows that $n^2 > 9$.

Fallacy: Affirming the Conclusion

$(q \wedge (p \rightarrow q)) \rightarrow p$ is a **contingency**. It states that if we know that both an implication $p \rightarrow q$ is true and that its conclusion, q, is true, then the hypothesis, p, is true.

Ex: Suppose the implication "If the bus breaks down, then I will have to walk" and its conclusion "I will have to walk" is true. It **does not** follow that the bus broke down. Perhaps I simply missed the bus.

Ex: Consider the implication $(n > 3) \rightarrow (n^2 > 9)$ which is true. Suppose also that $n^2 > 9$. It does not follow that $n > 3$. It might be that $n = -4$ for example.