Problem S22 (Signals and Systems)

Solution:

I used Mathematica to find some of the integrals, although you could use tables or integrate by parts.

(a)

$$\bar{t} = \int t g^2(t)\, dt = \int_0^\infty t^7 e^{-2t/tau}\, dt = \frac{315}{16}\tau^8$$

$$\bar{t} = \int g^2(t)\, dt = \int_0^\infty t^6 e^{-2t/tau}\, dt = \frac{45}{8}\tau^7$$

Therefore,

$$\bar{t} = \frac{7}{2}\tau$$

(b)

$$\int (t - \bar{t})^2 g^2(t)\, dt = \frac{315}{32}\tau^9$$

Therefore,

$$\Delta t = \sqrt{7}\,\tau$$

(c)

$$\int \dot{g}^2(t)\, dt = \frac{9}{8}\tau^5$$

Therefore,

$$\Delta\omega = \frac{2}{\sqrt{5}\,\tau}$$

(d) The duration-bandwidth product is

$$\Delta t\, \Delta\omega = 2\sqrt{\frac{7}{5}} \approx 2.366$$

which compares favotably with the theoretical lower bound

$$\Delta t\, \Delta\omega \geq 2$$