Unified Engineering II **Spring 2004**

Problem S13 (Signals and Systems)

In class, you learned about a *smoother*, with transfer function

$$G_1(s) = \frac{-a^2}{(s-a)(s+a)}$$

The smoother is an example of a *low-pass filter*, which means that it tends to attenuate high-frequency sine waves, but "pass" low-frequency sine waves. Unfortunately, the smoother is non-causal, which means that it can't be implemented in real time. A similar causal low-pass filter is

$$G_2(s) = \frac{a^2}{(s+a)^2}$$

In this problem, you will compare these two low-pass filters, to see how they affect sinusoidal inputs. Consider an input signal

$$u(t) = \cos \omega t$$

1. Find the transfer function, $G_1(j\omega)$, as a function of frequency, ω.

2. Since the transfer function is complex, it can be represented as

   $$G_1(j\omega) = A_1(\omega)e^{j\phi_1(\omega)}$$

   where the amplitude of the transfer function is $A_1(\omega)$, and the phase of the trasnfer function is $\phi_1(\omega)$. Find $A_1(\omega)$ and $\phi_1(\omega)$.

3. Find the transfer function, $G_2(j\omega)$, as a function of frequency, ω, as well as $A_2(\omega)$ and $\phi_2(\omega)$.

4. For the input $u(t)$ above, show that the output of the system G_1 is

   $$y_1(t) = A_1(\omega)\cos(\omega t + \phi_1(\omega))$$

   and do likewise for system G_2.

5. A_1 and A_2 determine how much the magnitude of the input cosine wave is affected by each filter. Ideally, A_1 and A_2 would be 1, meaning that the filters don't change the magnitude of the input sine at all. Which filter (if either) changes the magnitude the least?

6. ϕ_1 and ϕ_2 determine how much the phase of the input cosine wave is affected by each filter. Non-zero values of ϕ correspond to a shifting left or right (*i.e.*, advancing or delaying) the sine wave. Ideally, ϕ_1 and ϕ_2 would be zero, meaning that the filters don't change the phase of the input sine at all. Which filter (if either) produces the least phase shift?

7. Explain why the non-causal filter is preferred in signal processing applications where it can be applied.