

## Problem S12 (Signals and Systems)

The longitudinal dynamics of an aircraft flying are given in state-space form as

$$\frac{dh}{dt} = V \sin \gamma \quad (1)$$

$$\frac{dV}{dt} = -g \sin \gamma + \frac{T}{m} - \frac{D}{m} \quad (2)$$

$$\frac{d\gamma}{dt} = \frac{L - mg}{mV} \quad (3)$$

where

$h$  = altitude

$V$  = velocity of aircraft

$\gamma$  = flight path angle

$g$  = acceleration due to gravity

$L$  = lift

$T$  = thrust

$D$  = drag

$m$  = mass of aircraft

We assume that (1) the thrust is constant, (2) the aircraft flies at constant coefficient of lift, and (3) the aircraft flies at constant drag coefficient. These assumptions are not bad for the case when the pilot releases all the controls, and makes no adjustments, say, for altitude variations. In this case, we can rewrite the equations as

$$\frac{dh}{dt} = V \sin \gamma \quad (4)$$

$$\frac{dV}{dt} = -g \sin \gamma + \frac{T}{m} - \frac{D_0}{m} \frac{V^2}{V_0^2} \quad (5)$$

$$\frac{d\gamma}{dt} = \frac{L_0(V/V_0)^2 - mg}{mV} \quad (6)$$

where  $L_0$  and  $D_0$  are the lift and drag at the nominal velocity,  $V_0$ . If the aircraft is initially in trim, it must also be true that

$$T = D_0 \quad (7)$$

$$L_0 = mg \quad (8)$$

Equations (4)–(6) are the equations of motion in state-space form, although the equations are nonlinear. The equations may be *linearized*, by considering small perturbations  $\delta h$ ,  $\delta V$ , and  $\delta \gamma$  about the nominal trajectory. The linearized equations of

motion (after a little work) are

$$\frac{d}{dt} \delta h = V_0 \delta \gamma \quad (9)$$

$$\frac{d}{dt} \delta V = -g \delta \gamma - \frac{2g}{L_0/D_0} \frac{1}{V_0} \delta V \quad (10)$$

$$\frac{d}{dt} \delta \gamma = \frac{2g}{V_0^2} \delta V \quad (11)$$

So  $\dot{\underline{x}} = A\underline{x}$ , where

$$\underline{x} = \begin{bmatrix} \delta h \\ \delta V \\ \delta \gamma \end{bmatrix} \quad (12)$$

and

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -\frac{2g}{L_0/D_0} \frac{1}{V_0} & -g \\ 0 & \frac{2g}{V_0^2} & 0 \end{bmatrix} \quad (13)$$

For a light aircraft flying at a nominal velocity of 120 knots, with a lift-to-drag ratio of 10,

1. Find the eigenvalues and eigenvectors of the system.
2. For an initial perturbation

$$\delta h(0) = 0 \text{ m}$$

$$\delta V(0) = 0 \text{ m/sec}$$

$$\delta \gamma(0) = 0.1 \text{ rad}$$

find and plot the perturbations  $\delta h(t)$ ,  $\delta V(t)$ , and  $\delta \gamma(t)$  as functions of time. This perturbation corresponds to a perturbation caused by a vertical gust which puts the aircraft momentarily into a 0.1 rad (6 deg) climb.

Note: Some of the eigenvalues and eigenvectors are complex. As in the notes, treat a complex eigenvalue / eigenvector just as you would a real eigenvalue / eigenvector. You may find it useful to use Matlab, a spreadsheet, or Ada to calculate the results numerically.