Unified Engineering II **Spring 2004**

Problem S11 (Signals and Systems)

Consider an aircraft flying in cruise at 250 knots, so that

$$v_0 = 129 \text{ m/s}$$

Assume that the aircraft has lift-to-drag ratio

$$\frac{L_0}{D_0} = 15$$

Then the transfer function from changes in thrust to changes in altitude is

$$G(s) = \frac{2g}{mv_0} \frac{1}{s\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)} \tag{1}$$

where the *natural frequency* of the phugoid mode is

$$\omega_n = \sqrt{2}\frac{g}{v_0} \tag{2}$$

the *damping ratio* is

$$\zeta = \frac{1}{\sqrt{2}(L_0/D_0)} \tag{3}$$

and $g = 9.82$ m/s is the acceleration due to gravity. The transfer function can be normalized by the constant factor $\frac{2g}{mv_0}$, so that

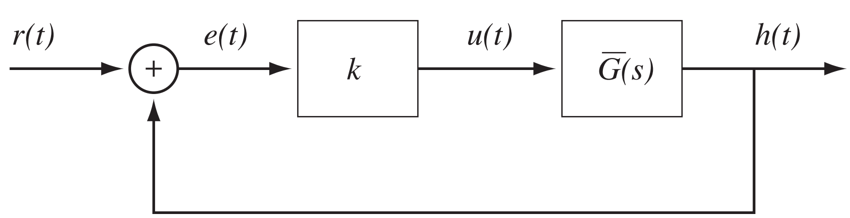
$$\bar{G}(s) = \frac{1}{s\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)} \tag{4}$$

is the normalized transfer function, corresponding to normalized input

$$u(t) = \frac{2g}{mv_0}\delta T$$

1. Find and plot the impulse response corresponding to the transfer function $\bar{G}(s)$, using partial fraction expansion and inverse Laplace techniques. Hint: The poles of the system are complex, so you will have to do complex arithmetic.

2. Suppose we try to control the altitude through a feedback loop, as shown below



That is, the control input $u(t)$ (normalized throttle) is a gain k times the error, $e(t)$, which is the difference between the altitude $h(t)$ and the altitude reference $r(t)$. The transfer function from $r(t)$ to $h(t)$ can be shown to be

$$H(s) = \frac{1}{1 + kG(s)}$$

For the gain k in the range $[0, 0.1]$, plot the poles of the system in the complex plane. You should find that for any positive k, the complex poles are made less stable. What gain k makes the complex poles unstable, i.e., for what gain is the damping ratio zero?

3. For the gain k in the range $[-0.1, 0]$, plot the poles of the system in the complex plane. You should find that for any negative k, the real pole is unstable.

Note that neither positive gain or negative gain makes the system more stable than without feedback control. It is possible to do better with a dynamic gain, but this problem should give you an idea of why the phugoid dynamics are so hard to control with throttle only.