Unified Engineering II **Spring 2004**

Problem S11 (Signals and Systems) Solution

1. From the problem statement,

$$\omega_n = \sqrt{2}\ \frac{9.82\ \text{m/s}^2}{129\ \text{m/s}} = 0.1077\ \text{r/s}$$
$$\zeta = \frac{1}{\sqrt{2}(L_0/D_0} = \frac{1}{\sqrt{2}\cdot 15} = 0.0471$$

Therefore,

$$\bar{G}(s) = \frac{1}{s\left(s^2 + 0.01015s + 0.0116\right)}$$

The roots of the denominator are at $s = 0$, and

$$s = \frac{-0.01915 \pm \sqrt{0.01015^2 - 4\cdot 0.0116}}{2}$$
$$= -0.005075 \pm 0.1075j$$

So

$$\bar{G}(s) = \frac{1}{s\left(s - [-0.005075 + 0.1075j]\right)\left(s - [-0.005075 - 0.1075j]\right)}$$

Use the coverup method to obtain the partial fraction expansion

$$\bar{G}(s) = \frac{86.283}{s} + \frac{-43.142 + 2.036j}{s - [-0.005075 + 0.1075j]} + \frac{-43.142 - 2.036j}{s - [-0.005075 - 0.1075j]}$$

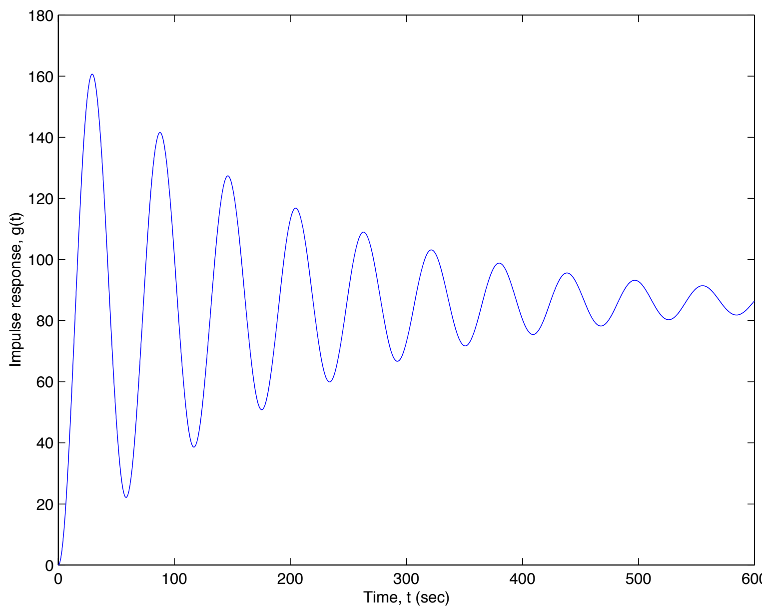
Taking the inverse Laplace transform (assuming that $\bar{g}(t)$ is causal), we have

$$\bar{g}(t) = 86.283\sigma(t) + (-43.142 + 2.036j)e^{(-0.005075+0.1075j)t} + (-43.142 - 2.036j)e^{(-0.005075-0.1075j)t}$$

Therefore,

$$\bar{g}(t) = \sigma(t)\left[86.283 + 2e^{-0.005075t}\left(-43.142\cos\omega_d t - 2.036\sin\omega_d t\right)\right]$$
$$= \sigma(t)\left[86.283 + \left(-86.284\cos\omega_d t - 4.072\sin\omega_d t\right)e^{-0.005075t}\right]$$

where $\omega_d = 0.1075$ r/s. See below for the impulse response.



2. From the problem statement,

$$\frac{H(s)}{R(s)} = \frac{k\bar{G}(s)}{1+k\bar{G}(s)}$$

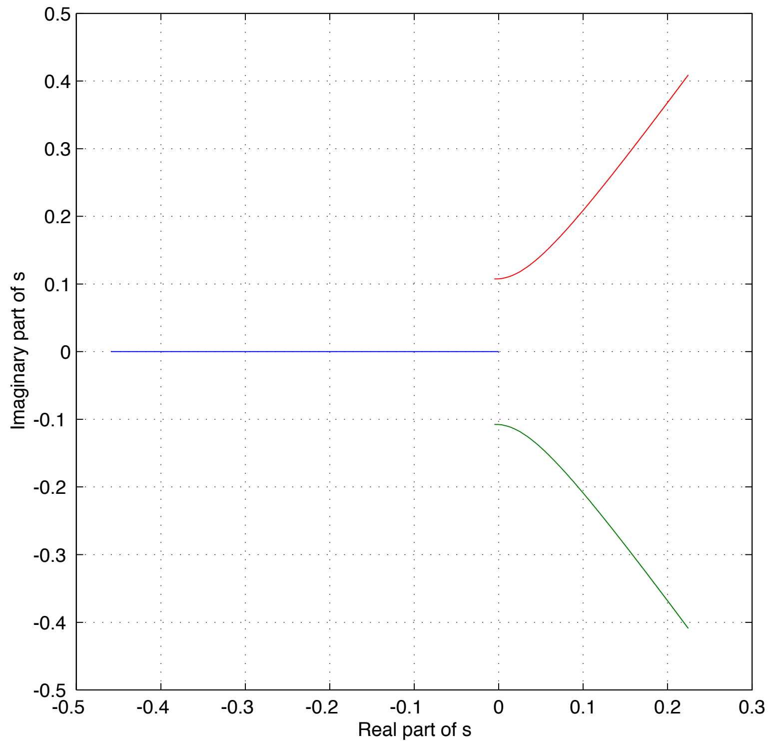
$$= \frac{k\,\dfrac{1}{s(s^2+2\zeta\omega_n s+\omega_n^2)}}{1+k\,\dfrac{1}{s(s^2+2\zeta\omega_n s+\omega_n^2)}}$$

$$= \frac{k}{s^3+2\zeta\omega_n s^2+\omega_n^2 s+k}$$

So the poles of the system are the roots of the denominator polynomial,

$$\phi(s) = s^3 + 2\zeta\omega_n s^2 + \omega_n^2 s + k = 0$$

The roots can be found using Matlab, a programmable calculator, etc. The plot of the roots (the "root locus") is shown below. Note that the oscillatory poles go unstable at a gain of only $k = 0.000118$.



3. The roots locus for negative gains can be plotted in a similar way, as below. Note that the real pole is unstable for all negative k.