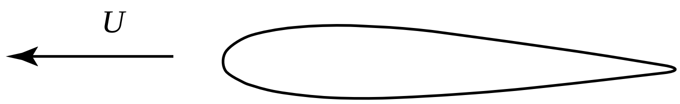
Unified Engineering II **Spring 2004**

Problem S3 (Signals and Systems)

Note: Please do not use official or unofficial bibles for this problem.

An airfoil with chord c is moving at velocity U with zero angle of incidence through the air, as shown in the figure below:



The air is not motionless, but rather has variations in the vertical velocity, w. As the airfoil flies through this gust field, the leading edge of the airfoil "sees" a variation in the angle of attack. If w is small compared to U, then the angle of attack change seen by the airfoil is $\alpha = w/U$. Since the velocity profile varies in space, the angle of attack seen by the airfoil is a function of time, $\alpha(t)$.

One might expect that the lift coefficient of the airfoil is just

$$C_L(t) = 2\pi\alpha(t)$$

However, the airfoil does not respond instantaneously as the airfoil encounters the gust. If the airfoil encounters a "sharp-edged gust," so that the apparent change in the angle of attack is a step function in time,

$$\alpha(t) = \alpha_0\sigma(t)$$

then the change in lift is given by

$$C_L(t) = 2\pi\alpha_0\psi(\bar{t})$$

where $\bar{t} = 2Ut/c$ is the dimensionless time. $\psi(\bar{t})$ is the *Küssner function*, and is the step response of the airfoil (neglecting multiplicative constants), if the input is considered to be the vertical gust at the leading edge as a function of time, and the output is considered to be the lift as a function of time. The Küssner function can be approximated as

$$\psi(\bar{t}) = \begin{cases} 0, & \bar{t} < 0 \\ 1 - \frac{1}{2}e^{-0.13\bar{t}} - \frac{1}{2}e^{-\bar{t}}, & \bar{t} \geq 0 \end{cases}$$

Assuming that the airfoil acts as an LTI system, determine and plot the lift coefficient, $C_L(t)$, and the gust velocity, $w(t)$, for the following conditions:

$$\begin{aligned} c &= 1 \text{ m} \\ U &= 1 \text{ m/s} \\ w(t) &= \begin{cases} 0 \text{ m/s}, & t < 0 \text{ s} \\ 0.1 \cdot (1 - e^{-2t}) \text{ m/s}, & t \geq 0 \text{ s} \end{cases} \end{aligned}$$