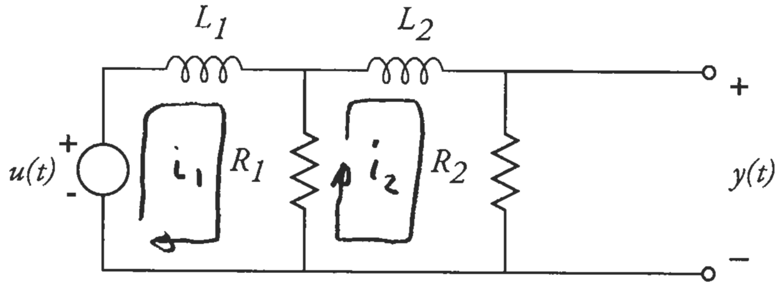
1. Find and plot the step response of the system



where $L_1 = L_2 = 2$ H, $R_1 = 2\ \Omega$, and $R_2 = 3\ \Omega$.

First, use the loop method to write d.e. for system:

i_1 loop: $$L_1 \frac{di_1}{dt} + R_1 i_1 - R_2 i_2 = u = \Gamma(t)$$

i_2 loop: $$-R_2 i_1 + L_2 \frac{di_2}{dt} + (R_1 + R_2) i_2 = 0$$

To find step response, (1) find homogeneous solution; (2) find particular solution; (3) add, and set constants to match initial conditions.

<u>Homogeneous Solution</u>

To find homogeneous solutions, assume $i = Ie^{st}$, and set right hand side to zero. Then

$$\underbrace{\begin{bmatrix} L_1 s + R_1 & -R_2 \\ -R_2 & L_2 s + R_1 + R_2 \end{bmatrix}}_{M(s)} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

In terms of the component values,

$$M(s) = \begin{bmatrix} 2s + 2 & -2 \\ -2 & 2s + 5 \end{bmatrix}$$

The characteristic values are given by

$$0 = \det[M(s)]$$
$$= (2s+2)(2s+5) - (-2)(-2)$$
$$= 4s + 14s + 6$$

The roots of this equation are

$$s_1 = -0.5, \quad s_2 = -3$$

The characteristic vectors are then solved for:

<u>$s_1 = -0.5$:</u>

$$M(-0.5) = \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$$

$$\Rightarrow \underline{\underline{I}} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad \text{(or any multiple)}$$

<u>$s_2 = -3$:</u>

$$M(-3) = \begin{bmatrix} -4 & -2 \\ -2 & -1 \end{bmatrix}$$

$$\Rightarrow \underline{\underline{I}} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \quad \text{(or any multiple)}$$

Therefore, the general homogeneous solution is

$$\begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} = a \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-0.5t} + b \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-3t}$$

<u>Particular solution</u>

To find the particular solution, set $u = 1$, and assume solution is a constant. Then a particular solution satisfies

$$\begin{bmatrix} +R_1 & -R_2 \\ -R_2 & R_1 + R_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

with component values,

$$\begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Solving,

$$i_1(t) = 5/6 \,, \quad i_2(t) = 1/3$$

Total Solution

The total solution is the sum of the homogeneous and particular solutions:

$$\begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} = a\begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-0.5t} + b\begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-3t} + \begin{bmatrix} 5/6 \\ 1/3 \end{bmatrix}$$

$$t \geq 0$$

The initial conditions are

$$i_1(0) = i_2(0) = 0$$

Since the initial current through the inductors is zero. Therefore,

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = a\begin{bmatrix} 2 \\ 1 \end{bmatrix} + b\begin{bmatrix} 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 5/6 \\ 1/3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -5/6 \\ -1/3 \end{bmatrix}$$

$$\Rightarrow \quad a = -2/5 \,, \quad b = -1/30$$

Therefore, the total solution is

$$\begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} = -\frac{2}{5}\begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-0.5t} - \frac{1}{30}\begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-3t} + \begin{bmatrix} 5/6 \\ 1/3 \end{bmatrix}$$

$$t \geq 0$$

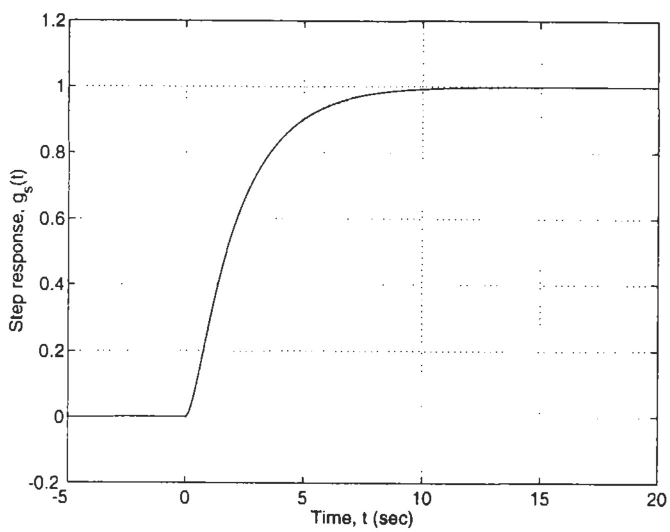
Finally, the output is given by

$$y(t) = R_2 i_2(t) = 3 i_2(t)$$

Therefore,

$$\boxed{g_s(t) = \begin{cases} 1 - \frac{6}{5} e^{-0.5t} + \frac{1}{5} e^{-3t}, & t \geq 0 \\ 0, & t < 0 \end{cases}}$$

This can be plotted using, say, Matlab or Excel:



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2. For the input signal

$$u(t) = \begin{cases} 0, & t < 0 \\ 2, & 0 \leq t < 1 \\ -1, & t \geq 1 \end{cases}$$

find and plot the output $y(t)$, using superposition.

Note that $u(t)$ is a sum of steps:

$$u(t) = 2\,\sigma(t) - 3\,\sigma(t-1)$$

Therefore,

$$y(t) = 2\, g_s(t) - 3\, g_s(t-1)$$

$$= 0 \qquad (t < 0)$$

$$= 2 - \frac{12}{5} e^{-0.5t} + \frac{2}{5} e^{-3t} \qquad (0 \le t < 2)$$

$$= 2 - \frac{12}{5} e^{-0.5t} + \frac{2}{5} e^{-3t} - 3 + \frac{18}{5} e^{-0.5(t-1)} - \frac{3}{5} e^{-3(t-1)} \qquad (t \geq 2)$$

Again, this can be plotted in Matlab or Excel: