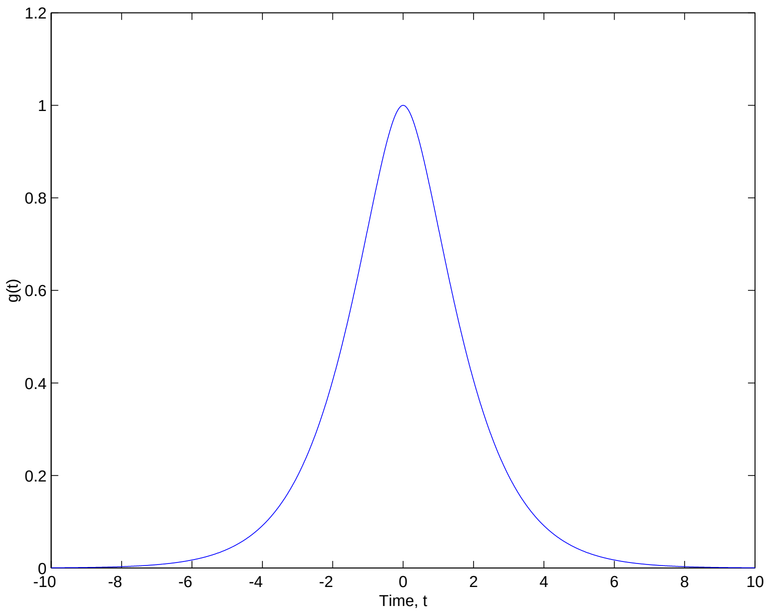
Problem S21 (Signals and Systems)

Solution:

1. The signal is plotted below:



The signal is very smooth, almost like a Gaussian. Therefore, I expect that the duration bandwidth product will be close to the theoretical lower bound.

2.
$$\left(\frac{\Delta t}{2}\right)^2 = \frac{\int t^2 g^2(t)\,dt}{\int g^2(t)\,dt}$$

The two integrals are easily evaluated for the given $g(t)$. The result is

$$\int t^2 g^2(t)\,dt = \frac{7}{2}$$

$$\int g^2(t)\,dt = \frac{5}{2}$$

Therefore,

$$\Delta t = 2\sqrt{\frac{7}{5}}$$

3. The time domain formula for the bandwidth is

$$\left(\frac{\Delta \omega}{2}\right)^2 = \frac{\int \dot{g}^2(t)\,dt}{\int g^2(t)\,dt}$$

The numerator integral is

$$\int \dot{g}^2(t)\,dt = \frac{1}{2}$$

Therefore,

$$\Delta\omega = \frac{2}{\sqrt{5}}$$

4. The duration-bandwidth product is

$$\Delta t \, \Delta\omega = \frac{4\sqrt{7}}{5} \approx 2.1166$$

which is very close to the theoretical lower limit of 2. This is not surprising, since the shape of $g(t)$ is close to a gaussian.

Problem S22 (Signals and Systems)

Solution:

I used Mathematica to find some of the integrals, although you could use tables or integrate by parts.

(a)

$$\bar{t} = \int t g^2(t)\,dt = \int_0^\infty t^7 e^{-2t/tau}\,dt = \frac{315}{16}\tau^8$$

$$\bar{t} = \int g^2(t)\,dt = \int_0^\infty t^6 e^{-2t/tau}\,dt = \frac{45}{8}\tau^7$$

Therefore,

$$\bar{t} = \frac{7}{2}\tau$$

(b)

$$\int (t-\bar{t})^2 g^2(t)\,dt = \frac{315}{32}\tau^9$$

Therefore,

$$\Delta t = \sqrt{7}\,\tau$$

(c)

$$\int \dot{g}^2(t)\,dt = \frac{9}{8}\tau^5$$

Therefore,

$$\Delta\omega = \frac{2}{\sqrt{5}\,\tau}$$

(d) The duration-bandwidth product is

$$\Delta t\,\Delta\omega = 2\sqrt{\frac{7}{5}} \approx 2.366$$

which compares favotably with the theoretical lower bound

$$\Delta t\,\Delta\omega \geq 2$$