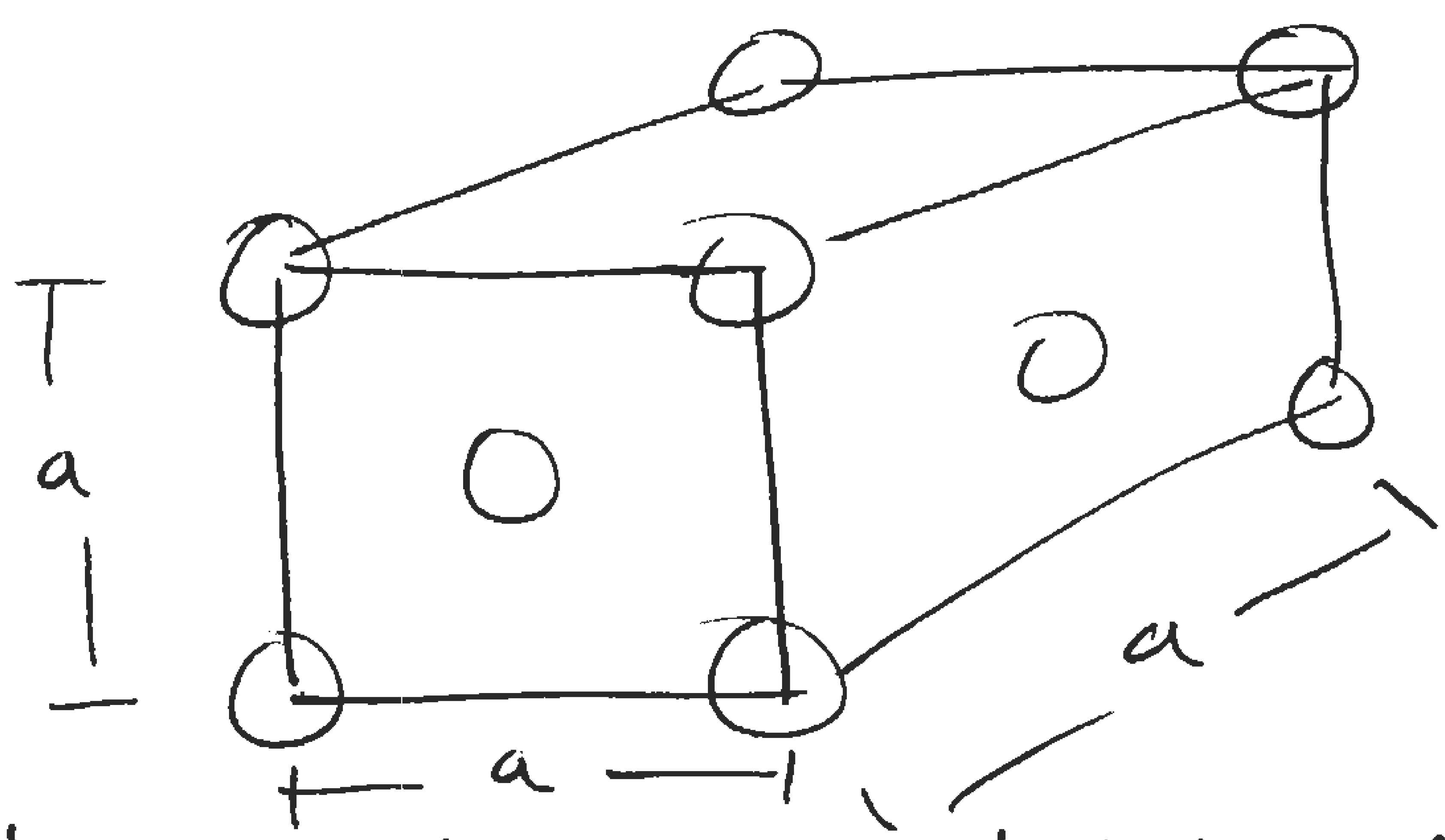
M24 FCC Packing density

- close packed directions on face diagonals

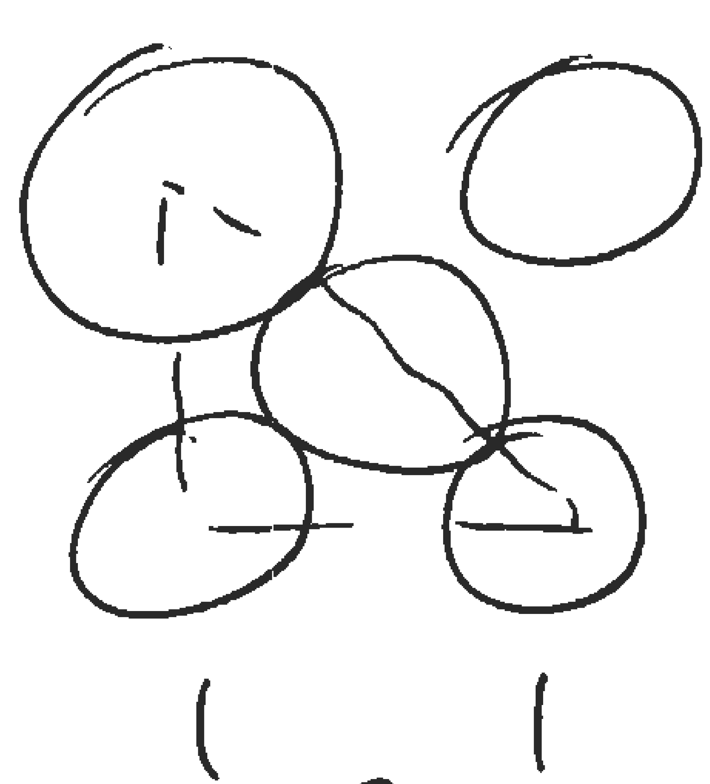


atomic radius r, atomic volume $\frac{4}{3}\pi r^3$

number of atoms / cube $= \underbrace{(8\times\frac{1}{8})}_{\text{corners}} + \underbrace{6\times(\frac{1}{2})}_{\text{faces}} = 4$

$\therefore$ $4\times\frac{4}{3}\pi r^3$ / cube. $= \frac{16}{3}\pi r^3$ of "solid"

side of cube =



$$2a^2 = (4r)^2 = 16r^2$$

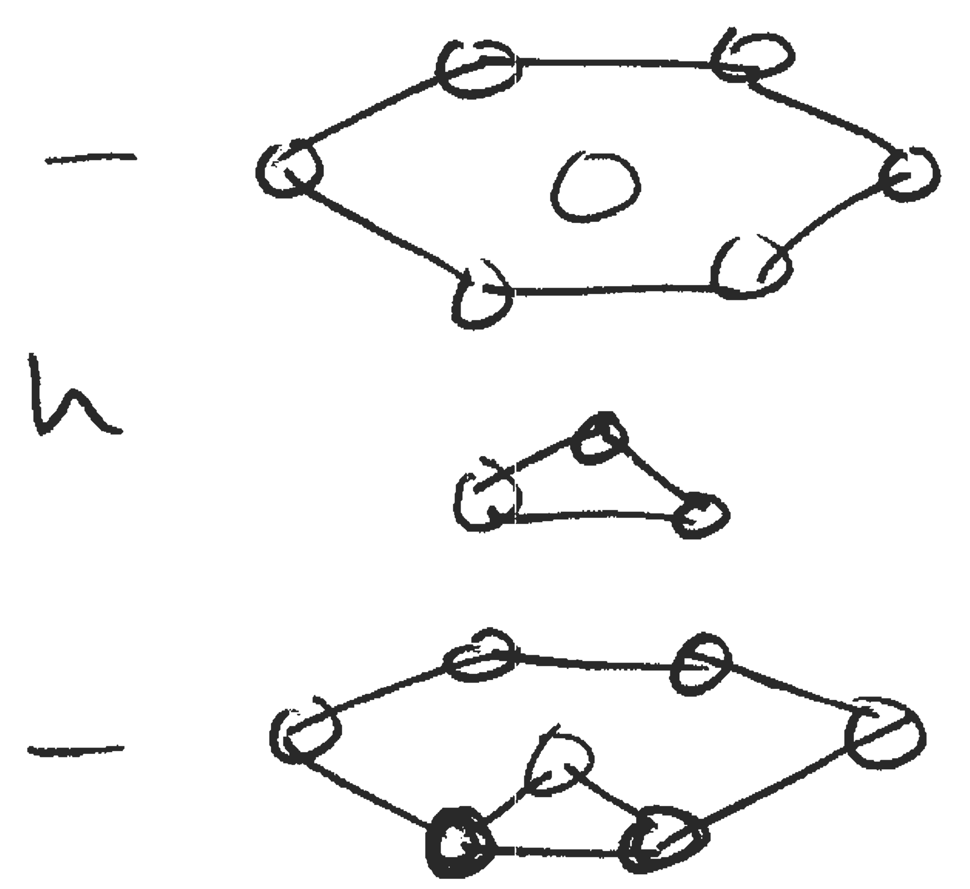
$$a = \sqrt{8}\, r$$

$\therefore$ packing density

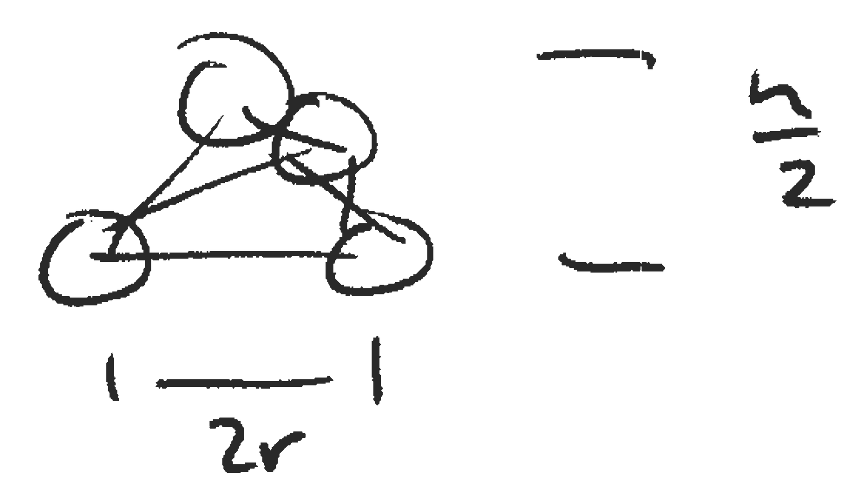
$$= \frac{16\pi \cancel{r^3}}{3} \Big/ (\sqrt{8})^3 \cancel{r^3} = \frac{16\pi}{3(\sqrt{8})^3} =$$

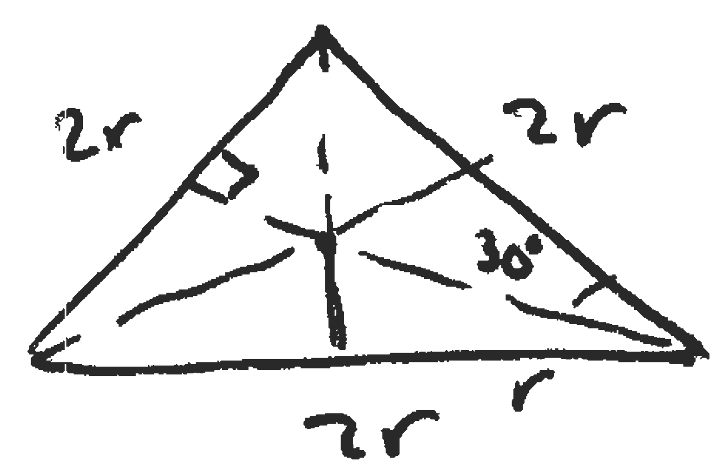
$$= 0.740$$

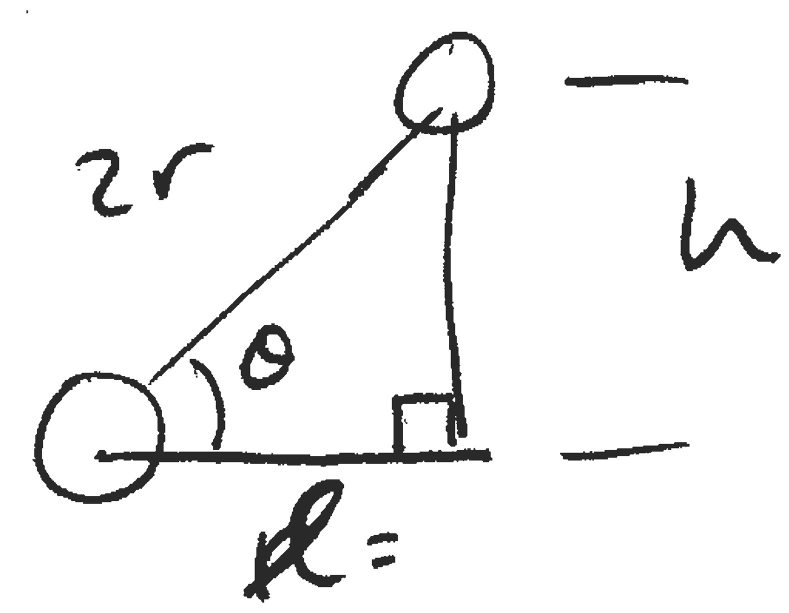
M24 HCP unit cell



consider tetrahedron







$$l\cos 30^\circ = r$$

$$l = \frac{2r}{\sqrt{3}}$$

$$\cos\theta = \frac{\cancel{2r}}{\sqrt{3}}\cdot\frac{1}{\cancel{2r}} = \frac{1}{\sqrt{3}}$$

$$h^2 = 4r^2 - \frac{4r^2}{3} = \frac{8}{3}r^2$$

area of triangle =

$$2r\times \cancel{2}r.\frac{\sqrt{3}}{\cancel{2}} = 2\sqrt{3}\,r^2$$

$$h = \sqrt{\frac{8}{3}}\,r$$

$\therefore$ volume of hexagonal unit cell $= 6\times 2\cancel{\sqrt{3}}\,r^2\times\sqrt{\frac{8}{\cancel{3}}}\,r$

$$= 12\sqrt{8}\,r^3$$

Number of atoms/cell $= \underbrace{2\times\frac{1}{2}}_{\text{top \& bottom faces}} + \underbrace{12\times\frac{1}{6}}_{\text{edge atoms}} + \underbrace{3\times 1}_{\text{centre atoms}} = 6$

$\therefore$ packing density $= \dfrac{\cancel{6}\times\frac{2\cancel{4}}{3}\pi \cancel{r^3}}{\cancel{12}\sqrt{8}\,\cancel{r^3}} = \dfrac{2\pi}{3\sqrt{8}} = 0.74$

6) Density $= \frac{\text{Mass}}{\text{Volume}}$

at unit cell level $= \frac{\text{\#atoms/cell}}{\text{volume of cell}}$ ~~× volume of~~

i) Nickel. FCC. Mass/atom $= \frac{58.69}{6.023\times10^{23}}$

volume of cell $= \frac{\frac{58.69}{6.023\times10^{23}}\times 4}{8.90\times\cancel{10^{3}}} = a^3.$

$a^3 = 4.38\times10^{-26}$, $a = 3.52\times10^{-10}$
$= 0.3.59$ nm ⇐

$a = \sqrt{8}\,r \Rightarrow r = 1.24\times10^{-9}$ m ⇐.

ii) For Magnesium, HCP Mass/atom $= \frac{24.31}{6.023\times10^{23}}$

volume of cell $= \frac{\frac{24.31\times 6}{\cancel{1.74\times10^{3}}\,6.023\times10^{23}}}{1.74\times\cancel{10^{3}}} = 6\times2\sqrt{8}\,r^3$

$r^3 = 4.1\times10^{-27}$ $r = 1.60\times10^{-10}$ m ⇐

$h = \sqrt{\frac{8}{3}}\,r = 2.61\times10^{-10}$ m ⇐