M15) i) consider element of initial sides 1 unit in length

deformed element has sides $1+\varepsilon_1$, $1+\varepsilon_2$, $1+\varepsilon_3$

$\therefore$ Volume (deformed) = $(1+\varepsilon_1)(1+\varepsilon_2)(1+\varepsilon_3)$

neglecting high order terms ($\varepsilon_1\varepsilon_2$, $\varepsilon_1\varepsilon_3$, $\varepsilon_2\varepsilon_3$, $\varepsilon_1\varepsilon_2\varepsilon_3$)

deformed volume = $1+\varepsilon_1+\varepsilon_2+\varepsilon_3$

$\therefore$ $\dfrac{V_{def} - V_{undef}}{V_{undef}} = \varepsilon_1+\varepsilon_2+\varepsilon_3$ ∈

ii)

iii a) $\varepsilon_{11} = \dfrac{\partial u_1}{\partial x_1} = (x_1 + 0.5x_2) \times 10^{-3}$

$\varepsilon_{22} = \dfrac{\partial u_2}{\partial x_2} = (0.5x_2 - x_1) \times 10^{-3}$

$\varepsilon_{33} = \dfrac{\partial u_3}{\partial x_3} = 0$

$\varepsilon_{12} = \frac{1}{2}\left(\dfrac{\partial u_1}{\partial x_2} + \dfrac{\partial u_2}{\partial x_1}\right) = \frac{1}{2}(-x_2 + 0.5x_1 + 0.5x_1 - x_2)$

$= \frac{1}{2}(x_1 - 2x_2) \times 10^{-3}$ ⟸

$\varepsilon_{23} = \frac{1}{2}\left(\dfrac{\partial u_2}{\partial x_3} + \dfrac{\partial u_3}{\partial x_2}\right) = 0$

$\varepsilon_{13} = \frac{1}{2}\left(\dfrac{\partial u_3}{\partial x_1} + \dfrac{\partial u_1}{\partial x_3}\right) = 0$

6) Rigid body rotation

$\Theta = \frac{1}{2}\left(\dfrac{\partial u_1}{\partial x_2} - \dfrac{\partial u_2}{\partial x_1}\right)$

$= \frac{1}{2}(-x_2 + 0.5x_1 - 0.5x_1 - x_2) \times 10^{-3}$

$= \frac{1}{2}(-2x_2) = x_2 \times 10^{-3}$ ⟸

c) at $x_1 = 5mm$, $x_2 = 7mm$.

$$\epsilon_{11} = \left(5 + \frac{7}{2}\right) \times 10^{-3} = 8.5 \times 10^{-3}$$

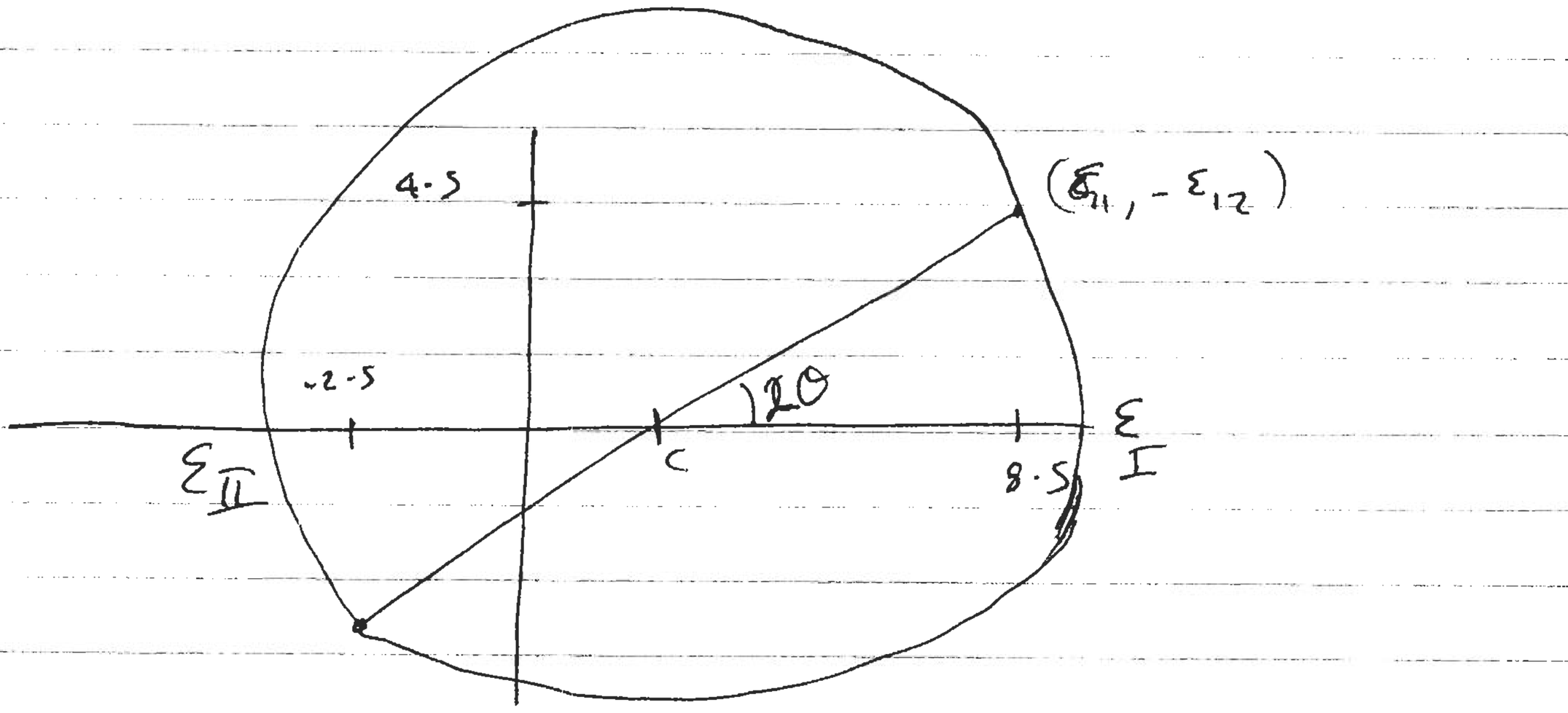
$$\varepsilon_{22} = \left(\frac{7}{2} - 5\right) \times 10^{-3} = -2.5 \times 10^{-3}$$

$$\varepsilon_{12} = \frac{1}{2}(5 - 14) \times 10^{-3} = -4.5 \times 10^{-3}$$

$$\varepsilon_{33} = 0$$

d) Volumetric strain $= (8.5 + (-2.5) + (0)) \times 10^{-3} = 1.5 \times 10^{-3}$ ⟸

c)



Center @ $\frac{1}{2}[8.5 + (-2.5)] \times 10^{-3} = +3 \times 10^{-3}$

Radius $= 10^{-3}\sqrt{(4.5)^2 + (5.5)^2} = 7.11 \times 10^{-3}$

@ $\varepsilon_I = 3 \times 10^{-3} + 7.11 \times 10^{-3} = 10.11 \times 10^{-3}$ ⟸

$\varepsilon_{II} = 3 \times 10^{-3} - 7.11 \times 10^{-3} = -4.11 \times 10^{-3}$ ⟸

$$\text{then } \theta = \tfrac{1}{2}\tan^{-1}\left(\frac{4.5}{5.5}\right) = 19.6^\circ \text{ clockwise from } x,$$

d) Volumetric strain =

$$10.11 + (-4.11) + 0 = 6 \times 10^{-3}$$