

a) $M_\infty = \frac{V_\infty}{a_\infty} = \frac{V_\infty}{\sqrt{\gamma RT_\infty}}$, but $RT = \frac{P}{\rho}$, so $a_\infty = \sqrt{\frac{\gamma P_\infty}{\rho_\infty}}$

$$M_\infty = V_\infty \sqrt{\frac{P_\infty}{\gamma \rho_\infty}}$$

b) $P_0 = P_\infty \left[1 + \frac{\gamma-1}{2} M_\infty^2 \right]^{\frac{\gamma}{\gamma-1}}$ exact

$$\begin{aligned} P_0 &= P_\infty + \frac{1}{2} \rho_\infty V_\infty^2 \\ &= P_\infty + \frac{1}{2} \rho_\infty M_\infty^2 \cdot \frac{\gamma P_\infty}{\rho_\infty} \end{aligned}$$

$$P_0 = P_\infty \left[1 + \frac{\gamma}{2} M_\infty^2 \right]$$

Bernoulli

Plot $\left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{\frac{\gamma}{\gamma-1}}$ and $1 + \frac{\gamma}{2} M_\infty^2$ attached

Plot $\left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{\frac{\gamma}{\gamma-1}} - \left(1 + \frac{\gamma}{2} M_\infty^2\right)$ attached

