

CP18-20

The problems in this problem set cover lecture [C17 = quiz review], C18, C19, C20

1. The operation \oplus is defined for two Boolean variables A, B as follows:

$$A \oplus B = \overline{A}B + A\overline{B}$$

Draw the truth table for $A \oplus B$

2. What are the minterms in the expression $A \oplus B \oplus C$?

Hint: Use a dummy variable D for $A \oplus B$, apply the Boolean algebra theorems, then replace D with $A \oplus B$ and repeat the process.

3. Convert the following English statements into formal propositions.
 - a. The killer touched both the candlestick and the wrench
 - b. There are exactly 2 sets of fingerprints on the candlestick.
 - c. Joe touched either the candlestick or the wrench, but not both.
 - d. George only touched the candlestick.
 - e. George saw Hannah touch the wrench.
 - f. Hannah touched all the weapons that George touched.
 - g. Hannah saw Joe touch the candlestick

Given that there is only one killer, use resolution to identify the killer.

4. Provide a **Direct Proof** of the following, where a, b, and c are integers

If $a|b$ and $b|c$, then $a|c$

Hint: definition of “ $|$ ” (Divisible) is given in lecture 20.

5. Prove using induction that $P(n) = P(n-1) + P(n-2)$, where $P(n)$ is a Fibonacci number.

Hint: What are Fibonacci numbers? That will help you identify the base case.

6. Prove using induction that if p does not divide any of the numbers $a_1, a_2, a_3, \dots, a_n$ (i.e., p is not a common divisor for $a_1, a_2, a_3, \dots, a_n$); then p does not divide $a_1 * a_2 * a_3 * \dots * a_n$